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DEALING WITH TECHNICAL PROBLEMS
RELATING TO THE PRODUCTS, PROCESSES AND INVESTIGATIONS OF
THE PHILIPS INDUSTRIES

A HIGH-SPEED SCANNING RADAR ANTENNA

by F. VALSTER.

621.396.677.7:621.396.965.

The considerable mass of conventional radar antennae and the wind forces acting on their large surface area, are a hindrance to the rapid scanning desirable for certain purposes. In the design discussed below, based on a principle due to Rinehart, only one small part of the antenna system rotates, making it possible to achieve very high speeds of revolution.

In plan-position radar it is frequently desirable to have a rapidly rotating beam. With conventional antenna constructions, however, this meets with difficulties due to the high moment of inertia of the rotating mass, which consists of a primary radiator (dipole or horn) combined with a reflector or lens. Further difficulties are occasioned by the varying wind forces on the large revolving surfaces.

Attempts have therefore been made to design radar antenna which dispense with the need for large rotating systems. The various methods devised to overcome this problem are all based on the Luneburg lens, which we shall presently discuss. An experimental antenna of this kind has also been built at Philips, with the object of studying the effect of the scanning speed on the radar picture.

In the following we shall first discuss some theoretical considerations relating to the speed of revolution of a radar beam, after which we shall describe the design of the present antenna and some results achieved.

Reasons for a high scanning speed

A high scanning speed in radar may be desirable for two reasons: firstly, to obtain a clear display of fast-moving objects, and secondly, for the sake of a bright radar picture. We shall examine these reasons in more detail.

Clear display of fast-moving objects

If the beam rotates slowly, a fast-moving object travels so far in the time taken by the antenna to complete one revolution that the radar echo on the P.P.I. screen is seen to move in a series of jumps. A jerky echo is difficult to distinguish from the irregular brightness variations due to noise, which are always present on the screen, and is also difficult to follow if there are a lot of permanent echoes. The latter was the case, for example, with one of our short-range radar sets, where the echoes of cars moving along a street were very hard to follow between the numerous permanent echoes from the street itself.

The optimum speed of revolution depends on the speed v of the observed object, on the radius R of the area covered on the radar screen, and on the radius r of the screen (possibly also on the diameter of the luminous spot). If d be the distance over which the echo of the object moves during one revolution of the beam and N the speed of revolution, the following relation holds:

$$N = \frac{r}{R}\frac{v}{d}. \qquad \dots\dots \quad (1)$$

The maximum value of d at which the irregular displacement of the echo is not yet troublesome is about 0.25 mm. If the screen has the usual diameter of 30 cm, it follows from (1) that:

$$N = \frac{v}{6R}\ \text{r.p.s.}, \qquad \dots\dots \quad (2)$$

where R is expressed in km and v in km/h. This relation is shown graphically in *fig. 1* for various values of R.

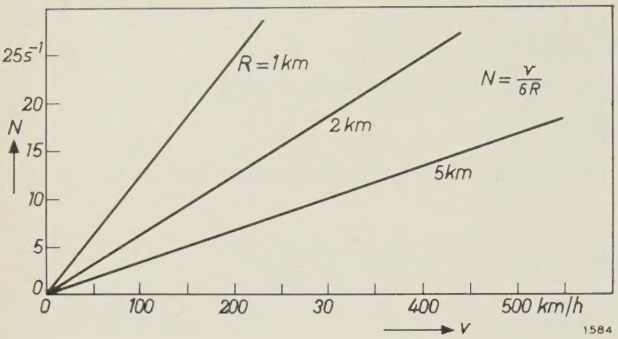

Fig. 1. The speed of revolution N of the beam for which the echo on a radar screen of 30 cm diameter moves 0.25 mm per revolution, is plotted as a function of the speed v of the object, for various values of the radius R of the area covered.

Bright radar picture

In some cases the radar picture is required to be bright enough to make it unnecessary to darken the room. Phosphors giving this brightness — and which are used in television picture tubes, for example — have a very short afterglow, however, and therefore the radar pattern must be traced many more times per second, i.e. fast scanning is necessary. The use of a television picture tube for radar would call for a minimum scanning speed of 20 r.p.s.

Operational consequences of high scanning speeds

A high scanning speed is not without its effects on the operation of the radar system. The faster the beam is rotated the more pulses must strike the objects per second if weak echoes are to be clearly distinguished from random noise fluctuations. In other words, a high scanning speed implies a high pulse-repetition frequency, and this involves difficulties with the modulator. Moreover, a high pulse repetition frequency f limits the radius R of the area covered, since a fresh pulse must not be sent out before the echoes of the last pulse have been received from objects at a distance R. This restricts the radius R to $c/2f$, where c is the speed of light.

The Luneburg lens

Scanning speeds of, say, 10 r.p.s. for small radar sets and 1 r.p.s. for large installations are evidently impracticable with conventional antenna systems — quite apart from the high power they would entail for the drive.

The solution of this problem is to be found — in principle at least — in a lens possessing rotational symmetry and which focusses the energy emitted by the primary radiator into the required beam. All that need then revolve (around the lens) is the primary radiator.

A familiar example of such a lens is the Luneburg lens ¹). This is spherical in shape and its refractive index n depends only on the distance r to the centre point, according to the following function:

$$\left.\begin{aligned} n(r) &= \sqrt{2-\left(\frac{r}{a}\right)^2} \quad \text{for} \quad r \leq a \\ n(r) &= 1 \quad\quad\quad\quad\quad \text{for} \quad r \geq a \end{aligned}\right\}, \quad . \quad (3)$$

where a is the radius of the lens. The refractive index thus changes continuously from $\sqrt{2}$ in the centre of the lens to 1 at the surface ($r = a$). A property of this lens is that every point O of its surface is imaged at infinity in the direction diametrically opposed to O (*fig. 2*). Rays entering the lens from O thus emerge as a parallel beam.

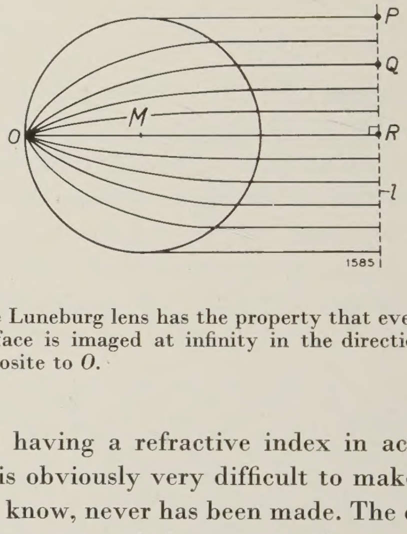

Fig. 2. The Luneburg lens has the property that every point O on its surface is imaged at infinity in the direction diametrically opposite to O.

A lens having a refractive index in accordance with (3) is obviously very difficult to make and, as far as we know, never has been made. The difficulty can be circumvented, however, with a two-dimensional analogue of the lens (i.e. one which forms a beam in one plane) such as described by Rinehart ²) and later calculated as one of a family of lenses of revolution by Toralda di Francia ³). Our lens, too, is an analogue of this type.

Let us take another look at fig. 2. In order to produce plane wave fronts l (perpendicular to the projection of the radius OM) from the spherical wave fronts issuing from O, the rays from O to the

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¹) R.K. Luneburg, Mathematical theory of optics, Brown University Press, Providence (Rhode Island, U.S.A.) 1944, p. 213.

²) R. F. Rinehart, A solution of the problem of rapid scanning for radar antennae, J. appl. Phys. **19**, 860-862, 1948. J. S. Hollis and M. W. Long, A Luneburg lens scanning system, Trans. Inst. Radio Engrs. **AP-5**, 21-25, 1957 (No. 1). For another solution, see G. D. M. Peeler and D. H. Archer, A two-dimensional microwave Luneburg lens, Trans. Inst. Radio Engrs. **AP-1**, 12-23, 1953 (No. 1).

³) G. Toralda di Francia, A family of perfect configuration lenses of revolution, Optica Acta **1**, 157-163, 1954/55.

straight line l must all, according to geometrical optics, have the same optical length. This is the case in the Luneburg lens, the rays passing through a medium of higher refractive index the shorter their geometrical length. The rays OP, OQ and OR thus have the same optical length, in other words the optical length

$$\int_O^l n \, ds$$

is independent of the path followed.

Paths of equal optical length between the point O and the line l are also produced if the rays can be given equal geometrical length and made to pass entirely through a medium with $n = 1$; in that case a medium with a refractive index varying according to (3) is not necessary. Thus, instead of letting the rays OQ and OR, for example, travel partly through an optically denser medium than OP, we let each of them make a detour such that OQ and OR get the same geometrical length as OP. If $n = 1$ along all rays, they have the same optical as well as the same geometrical length, and therefore l again forms a wave front. This is only possible, however, for beaming in two dimensions, the third dimension being required for the detour.

Form of the antenna

In the foregoing we have tacitly assumed that the energy can indeed be propagated along a curved surface. Assuming that this is possible (we shall return to this presently), we see that the lens from which we started becomes a sort of waveguide, which has the special property of converting circular wave fronts from a point source into linear wave fronts and has the advantage above the lens that it does not require a medium of varying refractive index.

The surface of the waveguide must meet the following requirements:

a) It must possess rotational symmetry.

b) All geodesics from a point on the circumference must have the same length from that point to a straight line perpendicular to the projection of the radius vector of the point; we can regard this line as the aperture of the antenna.
(A geodesic is the shortest superficial line between two points on any specified surface, and, according to Fermat's principle of least-time, this is the path along which the energy will propagate between those two points.)

The surfaces that satisfy these conditions can be found by the methods of differential geometry. *Fig. 3a* gives a sketch of such a surface, and fig. 3*b* a meridian cross-section of the surface. The equation for the meridian curve is expressed most simply by the coordinates ϱ and s, where ϱ is the direct distance from a point P to the axis of symmetry, and s is its distance *along* the meridian curve to the point A (fig. 3*b*). The equation is then (see the article by Rinehart, footnote [2])):

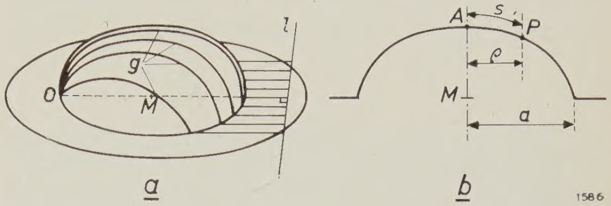


Fig. 3. *a*) In the hat-shaped surface illustrated, all geodesics g connecting the point O to the straight line l (perpendicular to OM) are of equal length. The surface also possesses rotational symmetry.
b) Meridian cross-section of the surface shown in (*a*).

$$s(\varrho) = \tfrac{1}{2}(\varrho + a \arcsin \varrho/a) \text{ for } 0 \leqq \varrho \leqq a. \qquad (4)$$

For $\varrho > a$ the surface defined by (4) gives way to a contiguous horizontal edge.

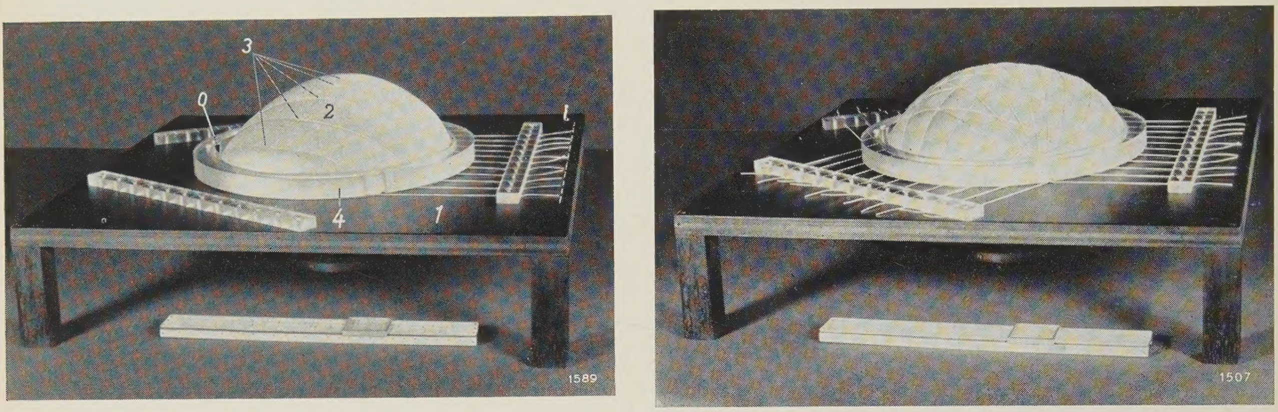
A model demonstrating the generation of the surface

Before going into the practical version of the antenna we shall discuss a model which demonstrates how a surface fulfilling the above-mentioned conditions can be generated. This model also helps to elucidate the points discussed above.

Fig. 4a shows a rectangular board *1* in which a circular hole is cut, of radius a. Fitted in this hole is an inflatable balloon *2*. A number of cords *3*, of equal length, are knotted together at one side, and the knot is fixed to a point O on the circumference of the hole. The free ends are then secured to the board in such a way that they lie on a straight line l which is perpendicular to the radius vector of point O (while this is done the balloon is not yet inflated and the cords are still slack). Fitted around the hole is a ring *4*, mounted in such a way that the cords can slide freely under it and at the same time are made to pass along the board from the balloon to the straight line l.

When the balloon is inflated, the cords are tensioned, and they slip into positions over the surface of the balloon such as to allow the balloon to expand as much as possible; in other words, each delineates its shortest path (the geodesic path) over the balloon. When this state has been reached, the balloon and the board together form a surface on which all geodesics from the point O to the straight line l are of equal length.

The circular hole by itself is insufficient to guarantee that the surface thus produced will possess rotational symmetry. To make it fulfil this condition, more than simply the one set of cords issuing from point O is required; other sets must be applied at points distributed around the circumference of the hole. Fig. 4*b* shows an arrangement using three sets of cords.



a *b*

Fig. 4. *a*) A model illustrating the generation of a surface on which all geodesics between a point O and a straight line l (perpendicular to the radius vector of O) are of equal length. *1* board with circular hole. *2* inflated balloon. *3* cords passing partly over the balloon and partly along the board, and which all have the same length from the point O to the straight line l. *4* ring, under which the cords pass freely and which holds the cords to the board surface between the balloon periphery and line l.
b) Three sets of cords are used here, producing a surface that better approaches rotational symmetry.

Construction of the antenna

We have still been working on the assumption that the energy *can* be propagated along a curved surface such as that in fig. 3. We shall now consider how this can be achieved, in other words, how a curved waveguide with the required profile can be constructed.

We shall consider first the familiar case of wave propagation between two parallel, flat plates of a material that is a good conductor. If we choose the distance $2d$ between the plates smaller than $\frac{1}{2}\lambda$, where λ represents the wavelength in free space, the energy has only one possible mode of propagation, viz. the TEM mode, in which the electrical field strength is perpendicular to the plates, and the velocity of propagation is equal to that in free space ($n = 1$, which is what we wanted).

We now apply the same principle to the curved surface. In order to compel the energy to propagate along the required lens surface, we use two metal plates which have virtually the same form as this surface, but are separated from it at either side by a distance d (measured perpendicular to the surface). If we again choose the distance $2d$ between the plates smaller than $\frac{1}{2}\lambda$, the deviation in our curved waveguide from the mode of vibration occurring in the flat case is negligible, provided the radius of curvature is everywhere large in relation to λ. This condition is easily fulfilled.

The question now is how the two metal plates can be given the correct shape. The first step is to calculate exactly the meridian curve of the geometrical surface possessing the above-mentioned properties (rotational symmetry, geodesics of equal length). For this purpose it is convenient to change from the coordinates ϱ and s in eq. (4) to rectangular coordinates. On the outside of the meridian thus found we now construct the curve that remains a constant distance d away from it (perpendicular to the meridian), and on the inside a curve which remains a constant distance $d + \delta$ therefrom, δ being the thickness of the sheet metal to be used, e.g. 1.5 mm. The curves produced are then the meridians (profiles) of the two dies in which two sheet metal plates can be forced into the correct shape (*fig. 5*).

To achieve reasonable accuracy with this graphic construction of the die profiles (deviation from the ideal profile nowhere more than e.g. 1% of the diameter $2a$) it is necessary to use meridian curves several times larger than the required dies. From a number of points on this enlarged meridian we then draw perpendiculars to the curve, and mark on each a distance d on the one side and a distance $d + \delta$ on the other. In this way we find the profiles of the dies. The coordinates of these profiles can also be computed by carrying out the above process analytically. This involves complicated calculations, which, without the help of an electronic computer, would be far too time-consuming .

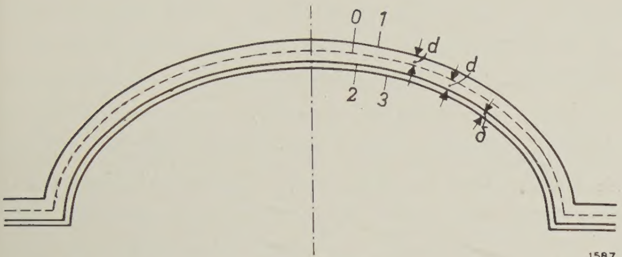


Fig. 5. Curve *0* is the calculated meridian curve of the geometrical surface possessing the required properties. Curves *1* and *2*, at a distance d ($< \frac{1}{4}\lambda$) at either side of *0*, are meridians of the metal surfaces to be made. Curve *3* lies at a further distance δ, equal to the thickness of the sheet metal used. *1* and *3* are the meridians of the jigs in which the metal sheeting will be stamped into the appropriate shape. (For clarity, the distances d and δ are exaggerated in comparison with the other dimensions.)

The two plates, shaped like hats, can be seen in *fig. 6*; the upper plate is marked with geodesics and wave fronts.

Further constructional details are illustrated schematically in *fig 7*. The two aluminium "hats" are kept at the right separation $2d$ by spacer blocks of "Polyfoam", a material whose dielectric constant differs very little from unity and thus behaves almost like air for the purposes of wave propagation. The brims of the two hats are not flat and parallel, but slightly tapered, giving a certain beaming effect in the vertical plane.

The whole assembly is supported from below by a central bushing *8* and from above by a tubular structure (*fig. 8a* and *b*). The rectangular waveguide *4* (fig. 7) is rotated by a small motor; the mouth of the waveguide constitutes the point source and revolves in the gap between the brims of the two hats. The fixed waveguide *7* connects the waveguide *4*

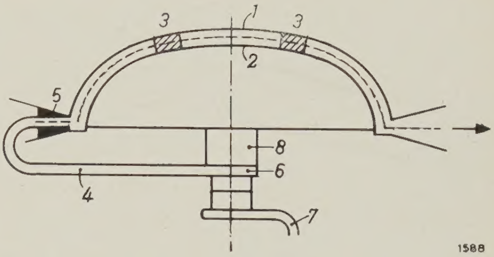


Fig. 7. Cross-section of the antenna system. *1* and *2* are the two hat-shaped plates, with spacers *3* of "Polyfoam". *4* rotating waveguide. *5* wedge-shaped block, preventing back radiation. *6* rotary joint between the rotating waveguide *4* and the fixed waveguide *7*, which connects the system to the transmitter and receiver. *8* bushing supporting the construction.

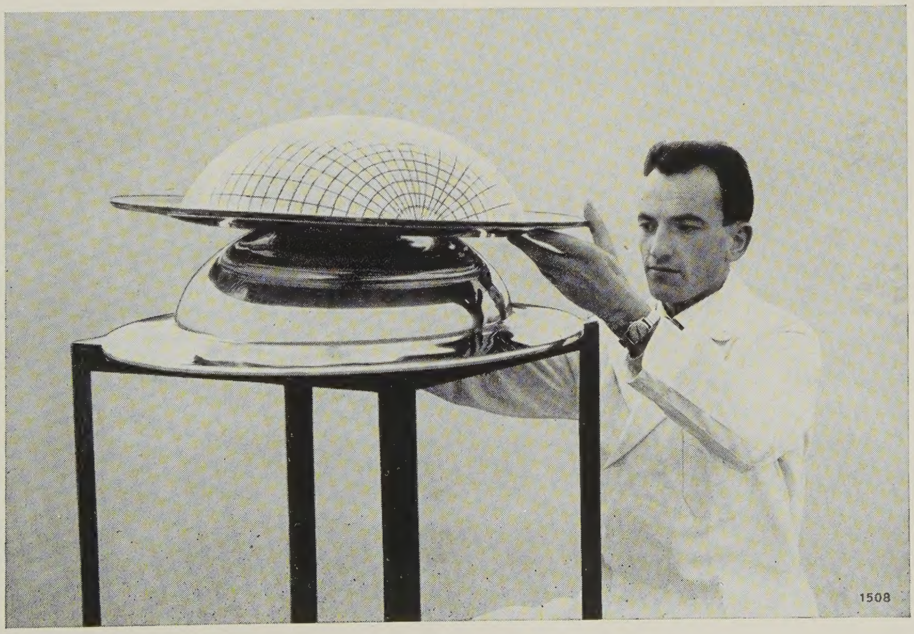


Fig. 6. The two hat-shaped surfaces, made from sheet aluminium. Geodesics and wave fronts are drawn on the upper surface.

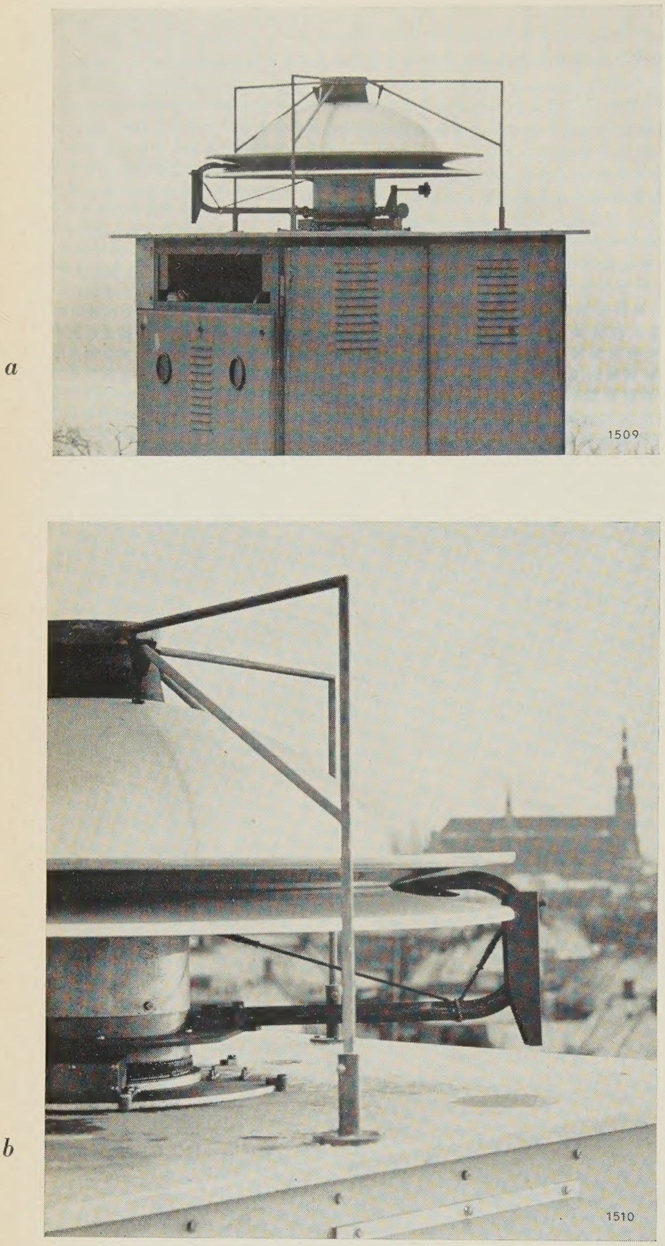


Fig. 8. *a*) Mounting of the antenna on the housing containing the radar transmitter and receiver.
b) Close-up view of antenna.

to the radar transmitter and receiver via the rotary coupling *6*. Back radiation is prevented by a wedge-shaped block *5* fixed around the mouth of waveguide *4*. It consists partly of aluminium with one or more grooves one quarter wavelength deep, functioning as a "choke", and partly of graphite-filled "Philite", which absorbs any residual radiation.

Performance

The antenna shown in fig. 8 forms part of an experimental radar installation operating on a wavelength of 1.25 cm. The pulses have a duration of 1/30 μs and a repetition frequency of 12 500 c/s. The diameter of the antenna (i.e. without the brim) is 50 cm; the distance $2d$ between the two "hats" is 0.5 cm, i.e. smaller than $\frac{1}{2}\lambda$.

The directional pattern in the horizontal plane shows a main lobe whose beam width Θ_h between the 3 dB directions, according to a rule of thumb, should be λ/D radians (D is the length of the antenna, in this case the length $2a$ of the antenna aperture); this rule of thumb, then, gives $\Theta_\mathrm{h} = 1.25/50$ radians $= 1.43°$. Since the hats show slight deviations from rotational symmetry, the directional pattern was found in fact to be somewhat dependent on the position of the energy feed. The average value of Θ_h was found by experiment to be 1.5°, which is thus in good agreement with the expected result.

The size of the side lobes depends on the radiation pattern of the primary radiator, which determines the amplitude distribution of the field in the antenna aperture. With the design chosen the level of the largest side lobes proved to be 18 dB below that of the main lobe, which is a sufficient difference for the purpose in view.

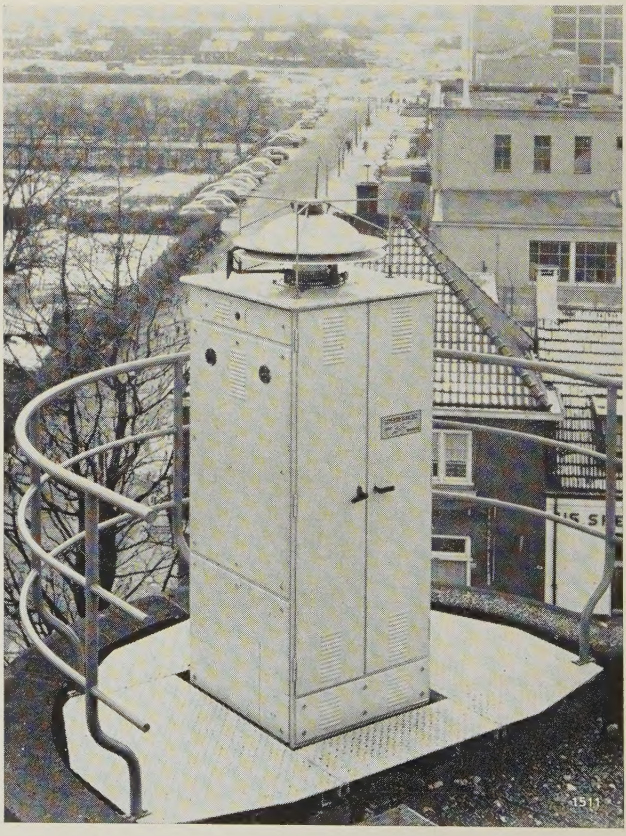


Fig. 9. Experimental radar installation with fast-scanning antenna system of the type described, on the roof of Philips Research Laboratories at Eindhoven.

The angle between the tapered brims of the hats is so chosen as to give the beam in the vertical plane an angular width $\Theta_V \approx 25°$.

The scanning speed of the antenna is variable up to a maximum of 10 revolutions per second. At the maximum speed the motor draws a power of only 200 W.

The radar system (*fig. 9*) using this antenna gives a display of an area in which streets with fairly heavy traffic can be seen. When the beam rotates at 10 r.p.s. the moving vehicles are indeed much more clearly distinguishable between the numerous permanent echoes than when the beam is rotated at the more usual speed of 1 or 2 r.p.s.

Summary. A high scanning speed (e.g. 10 r.p.s.) is desirable in plan-position radar when 1) the echo of fast-moving objects is to be followed clearly against a background of noise and numerous permanent echoes and 2) when a very bright radar picture is wanted, which is only realizable with short-persistence screens. Conventional radar antennae are not adapted to high-speed scanning. A rapidly revolving beam is obtainable, however, by using an analogue of the two-dimensional Luneburg lens, as described by Rinehart. This analogue is based on a geometrical property of a rotationally-symmetric hat-shaped surface whereby circular wave fronts in a given plane emanating from a point source are converted into linear wave fronts. The practical form of this idea consists of two metallic "hats", situated at a distance of less than one quarter wavelength on either side of the surface in question. The two hats constitute a waveguide. The point source is the mouth of a rectangular waveguide which revolves in the gap between the two brims; at the opposite side the radiation emerges in the form of a narrow beam. Some constructional details of an antenna of this type are given. In one version, for a wavelength of 1.25 cm, the main lobe has an average 3 dB beam width in the horizontal plane of 1.5°, and in the vertical plane of about 25°. A motor power of 200 W is sufficient for a speed of revolution of 10 r.p.s. The expectation that fast-moving objects are better observable at this speed than at e.g. 1 r.p.s., is confirmed.

In October 1959, Professor Dr. Balthasar van der Pol died at Wassenaar at the age of 70. In the years from 1922 to 1949, during which he acquired international repute in the field of radio science and engineering, he directed the work done in that field in Philips Research Laboratory at Eindhoven. In the same period he also played a prominent part in international organizational activities concerned with radio communications. After his retirement from industry he continued in these activities, principally as Director of the C.C.I.R. (Comité Consultatif International des Radiocommunications) at Geneva, an office he held until a few years before his death. Amongst the numerous distinctions conferred upon him were the Medal of Honor of the Institute of Radio Engineers, of which he was for some time vice-president, the Valdemar Poulsen Gold Medal, several honorary doctorates, and the honorary presidency of the U.R.S.I. (Union Radio Scientifique Internationale).

The obituary notices that appeared in journals both in the Netherlands and abroad were obviously unable to go far into the details of his scientific work. It gives the editors pleasure to be able to publish here a more comprehensive appreciation, written by Prof. Bremmer, one of Van der Pol's former and closest collaborators and probably the most familiar with his work. In our view such an appreciation is a fitting way of paying tribute to the memory of this remarkable man, and at the same time it gives the reader a glimpse of many important aspects of the evolution of radio science.

The title photograph shows Van der Pol (right) in conversation with Prof. Holst, the founder and first director of this laboratory.

THE SCIENTIFIC WORK OF BALTHASAR VAN DER POL

by H. BREMMER.

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In this article we shall try to review the many-sided scientific work done by the late Prof. Dr. Balthasar van der Pol, former member of the Philips Research Laboratories, Eindhoven, who died on the 6th October 1959. His numerous investigations were concerned primarily with problems of radio science and associated fields. The international name he soon acquired was due no doubt in the first place to his pioneering work on the *propagation of radio waves* and on *non-linear oscillations*. We shall see later how his study of these subjects led him to take an interest in many other problems, including problems of pure mathematics.

Van der Pol was one of those rare men who are equally at home in physics, engineering and mathematics. This enabled him to deal personally with the theoretical-mathematical treatment of technical problems, and also to meet professional mathematicians on their own ground and persuade them of the importance of a rigorous mathematical approach to such problems. He saw how difficult it often is to bring together the practitioners in these three fields. His success in this respect was a valuable facet of his life's work.

As a mathematician, Van der Pol showed a strong preference for heuristic methods, by which, along partly intuitive lines, results are readily arrived at that can only be verified later. To this extent he resembled Heaviside. This brilliant man, who lived from 1850 to 1925, used mathematical methods which proved to be very fruitful, but generally left it to others to demonstrate their validity. Heaviside was nevertheless the first to get to the bottom of such technical problems as the role of inductances in long-distance telephone cables. Van der Pol was a great admirer of this self-made scientific genius, as testified by his inaugural address on 8th December 1938 as extra-mural professor at Delft [1]). The address was entirely devoted to Heaviside, and underlined the significance of his work at a time when the application of Maxwell's equations was not yet a commonplace. The following sentence from Van der Pol's address will serve to illustrate the part played by Heaviside in the development of electrotechnical concepts that are now thoroughly familiar:

> "Electrotechnology and physics are also indebted to Heaviside for the concept and the word "impedance", not merely as understood in present-day alternating-current theory, but in a very much wider sense which includes on-off switching effects, a form which even today has still not been dealt with exhaustively."

In all his work Van der Pol owed much to the inspiration of Heaviside. This appears directly from his preoccupation with the methods of *operational calculus*, which were created by Heaviside for application to electrical networks. No one has explained better than Van der Pol the merits of operator methods. In particular he demonstrated their value for the discovery of numerous relations in widely diverse fields of mathematics. In one respect, however, there was a marked constrast between Heaviside and Van der Pol. The former suffered all his life from the difficulty of getting his ideas accepted, not least because of the almost illegible form of his writings. The latter's publications, on the other hand, were models of careful exposition and clarity of thought.

Van der Pol's lucidity was also much in evidence in the lectures he gave and in the innumerable meetings and discussions over which he presided. In this connection, mention must be made of his considerable talents as an organizer. The ease with which he could present a matter, and his complete familiarity with the whole field of radio, soon made him a leading figure in international organizations concerned with radio science and engineering, especially in the U.R.S.I. and the C.C.I.R. It was no surprise when, after retiring from Philips Research Laboratory, he was invited to be the first director of the C.C.I.R. upon the establishment of its permanent secretariat. In the time that he held this office (from 1949 to 1956) he worked with devotion to ensure that justice was done to pure scientific research in the advice issued on the preparation of technical regulations for international radio communications. It is scarcely believable that, with all his activities on international committees, he should still have found the time for abstract investigations. He succeeded in this only because his zest for work was quite out of the ordinary, and remained with him to the end of his life.

Van der Pol maintained that only a few great men have had a decisive bearing on the development of physics. He used to say that, faced with the enormous number of scientific investigations, it was easy to forget that the really fundamental work is contained in a mere handful of publications. He himself was therefore strongly drawn towards personal contact with the leading figures in the world of science. In that respect he was fortunate at the very beginning of his career. After taking his degree *cum laude* at Utrecht in 1916, he did experimental research in England until 1919. At first he worked in London under Fleming, the man who, in 1904, was granted the first patent on a diode. There followed a period of two years at Cambridge, where he worked under Sir J. J. Thomson. On his return to the Netherlands he was attached for three years to the Teyler Foundation at Haarlem, where he worked under Lorentz. Thus, Van der Pol was in close association with two scientists, one of whom he himself wittily described as the "discoverer" of the electron (in 1897 Thomson had determined the ratio e/m from electron orbits in a combined electrical and magnetic field) and the other as the "inventor" of the electron (in 1896 Lorentz had calculated e/m from the Zeeman effect). In the years that followed, Van der Pol was in close contact with other distinguished scientists, including Appleton, who was awarded the Nobel prize in 1947 for his pioneering research on the ionosphere.

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[1]) B. van der Pol, Oliver Heaviside (1850-1925), Ned. T. Natuurk. 5, 269-285, 1938.

Propagation of radio waves

In England Van der Pol soon came up against a problem which was to remain one of his great lifelong interests. We refer to the effect of the earth on the propagation of radio waves. In 1909 Sommerfeld [2]) had put forward a theory which explained the observed phenomena on the assumption that the earth's surface could be regarded as flat. As early as 1901, however, Marconi had succeeded in sending radio signals across the Atlantic Ocean. In 1915, after experience had been gained in the use of undamped waves, it had even proved possible to establish telephonic communication by radio across the Atlantic. It was evident that the curvature of the earth must have some effect on the propagation of the waves, because linear propagation straight through the highly absorbent earth would attenuate the signal beyond possibility of detection. It was conceivable that, as a result of diffraction, the waves might follow the surface of the earth far beyond the horizon as optically observed from the transmitting aerial. The existing theory on this idea was criticized by Poincaré, who showed in 1910 [3]) that, in the case of diffraction, the field over great distances must decrease proportionally to exp $(-\beta D/\lambda^{\frac{1}{3}})$, where λ is the wavelength and D the distance from the transmitter to the receiver measured over the earth's surface. Unfortunately, it was not at that time possible to test this theory, since there was no way of deriving a reliable numerical value for the constant β.

This was roughly the situation when Van der Pol first came into touch with the propagation problem. Unless the value of β was extremely small, the field upon diffraction would be so strongly attenuated that there could only be one other explanation for the long range of the radio waves: the presence of the ionosphere. The existance of conducting layers at high altitudes in the atmosphere, known as the ionosphere, was postulated in 1902 by Kennelly and Heaviside to account for the propagation of waves over great distances. The radio waves would then follow a zig-zag path, being reflected successively from the ionosphere and from the surface of the earth.

The mathematical difficulties of the pure diffraction problem, assuming the absence of the ionosphere, were bound up with the numerous Bessel functions occurring in the solution. This induced Van der Pol to encourage the mathematician Watson to study this problem, at a time when the latter was working on his well-known book on Bessel functions, which appeared in 1922. Watson's results, which were published in 1918 [4]), proved beyond doubt that the value of β was too large (it would be 0.00376 $\text{km}^{-\frac{2}{3}}$ if the earth were a perfect conductor and the atmosphere completely homogeneous) to explain the long range of radio waves without invoking the help of the ionosphere. The implications of Watson's calculations were discussed by Van der Pol in a paper published in 1919 [5]).

This did not mean, however, that further study of the pure diffraction problem, leaving the ionosphere out of account, was no longer necessary. In the first place the wave propagated along the earth, the "ground wave", makes an important contribution over short distances, particularly during the day, compared with the contribution from the "sky wave" which is propagated via the ionosphere. Moreover, the ionosphere has little effect on the extremely short waves used in television, frequency-modulated transmissions and radar. The further study of the ground wave was therefore still of importance. In cooperation with K. F. Niessen, Van der Pol elaborated on Sommerfeld's theory [6]), which could still be used for short distances, and made a special study of the influence of the height of the transmitting and receiving aerials above the ground. In later years the ground-wave theory was worked out by Van der Pol and the present author for distances at which the earth's curvature plays an essential part [7]). A mathematical method was developed for computing the fields of the ground wave for all topographical conditions. It was found that, even in the case of very short waves, the decrease of the field strength beyond the transmitter horizon was very much more gradual than had formerly been thought. This is illustrated in *fig. 1*, where the ratio of the actual field strength E on the earth's surface to the field strength E_{pr} in free space is plotted as a function of the distance from transmitter to receiver for a given case, viz. a transmitter at a height of 100 metres above the earth's surface and surface conditions corresponding to dry ground. The various curves show this ratio for four different wavelengths (differing successively from the other by a factor of 10) and for the limiting case $\lambda \rightarrow 0$. A distinct shadow effect, whereby the field strength is reduced by a factor of at least 4 within a distance of 5 km beyond the horizon, does not appear until the wavelength has dropped to about 7 mm.

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[2]) A. Sommerfeld, Ann. Physik **28**, 665, 1909.

[3]) H. Poincaré, Jahrb. drahtl. Telegr. Teleph. **3**, 445, 1910.

[4]) G. N. Watson, Proc. Roy. Soc. **A 95**, 83, 1918.

[5]) B. van der Pol, Phil. Mag. **38**, 365, 1919.

[6]) B. van der Pol and K. F. Niessen, T. Ned. Radiogen. **7**, 1, 1935.

[7]) B. van der Pol and H. Bremmer, Phil. Mag. **24**, 141 and 825, 1937.

The numerical results of the ground-wave propagation theory are of particular importance at wavelengths smaller than about 10 m, in which case the heights of the transmitting and receiving aerials above the earth's surface have a considerable nfluence. During Van der Pol's term of office with the C.C.I.R. a wide programme of computations was decided upon in order to get numerical data on ground-wave propagation, including the effect of refraction in the lowest layer of the atmosphere. The results were published in the form of graphs 8), an example of which is shown in *fig. 2*; this refers to a wavelength of 5 m, again for representative ground conditions (conductivity of 0.01 mho/m and a relative dielectric constant of 10). The graph indicates, for example, the extent to which the field increases with increasing height h_2 of the receiver for a fixed transmitter height h_1 of 10 m. The results are the same if the transmitter and receiver heights are interchanged.

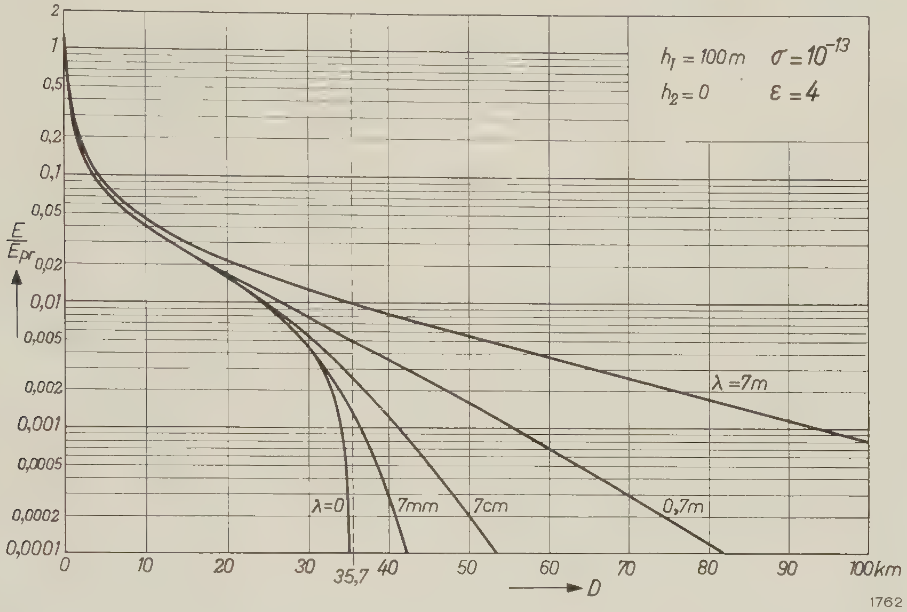

Fig. 1. Attenuation of a radio wave propagated along the curved surface of the earth (ground wave) for various wavelengths λ, calculated for dry ground and transmitter height h_1 = 100 m, receiver height h_2 = 0 m. The ratio of the field strength E on the earth to the field strength E_{pr} at the same distance in free space is plotted as a function of the distance D. A distinct shadow effect at the optical horizon (in this case at D = 35.7 km) appears only in the case of millimetre waves.

The rigorous theory on which such data are based is applicable to all problems of waves meeting a spherical obstacle. This situation also arises when light rays refracted in spherical drops of water give rise to rainbow effects. These considerations accordingly led to a new treatment of the rainbow in terms of wave theory 7) *).

We shall now return to those cases in which the sky wave conducted via the ionosphere is more important than the ground wave. As shown by Watson's calculations, mentioned above, this is the normal state in short-wave transmissions over great distances. The important role played by the ionosphere, established beyond all doubt by Watson's results, was later emphasized by Van der Pol in the following sentence from his above-mentioned inaugural speech:

> "You know of that conductive layer, high in the atmosphere, which makes radio communication over long distances possible, and whose existence is just as important to radio technology as the existence of iron in the earth is to electrical engineering as a whole".

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8) Atlas of ground-wave propagation curves for frequencies between 30 Mc/s and 300 Mc/s, C.C.I.R. Resolution No. 11, Geneva 1955.

*) *Editorial note:* By coincidence, a part of this wave treatment of the rainbow is to be seen scribbled on the blackboard in the title photograph.

Although the existence of the ionosphere was not definitely established until 1925, when direct reflection measurements placed it beyond all doubt, Eccles and Larmor had worked out a theory as early as 1912 for wave propagation through a conductive gas, which could be immediately applied to the ionosphere. It was found that a conductive gas behaves formally like a medium possessing a certain conductivity and a relative dielectric constant smaller than unity. This means that the phase velocity of radio waves in the ionosphere exceeds the speed of light c in a vacuum. However, the corresponding propagation velocity of discontinuities (group velocity) is smaller than c, as it must be to accord with the principles of the theory of relativity. By means of resonance measurements on lecher lines, Van der Pol determined, during his period in the Cavendish Laboratory at Cambridge, the conductivity and dielectric constant of the plasma in glow discharges; these experiments closely simulated the conditions in the ionosphere. After experimental difficulties had been overcome, it was possible in this way to measure for the first time an apparent dielectric constant smaller than unity. This represented experimental confirmation of the now generally accepted propagation mechanism for radio waves that are refracted in the ionosphere. This whole subject was clearly dealt with in Van der Pol's thesis in 1920, entitled "The influence of an ionized gas on the propagation of electromagnetic waves, and its applications in the field of wireless telegraphy and in measurements on glow discharges".

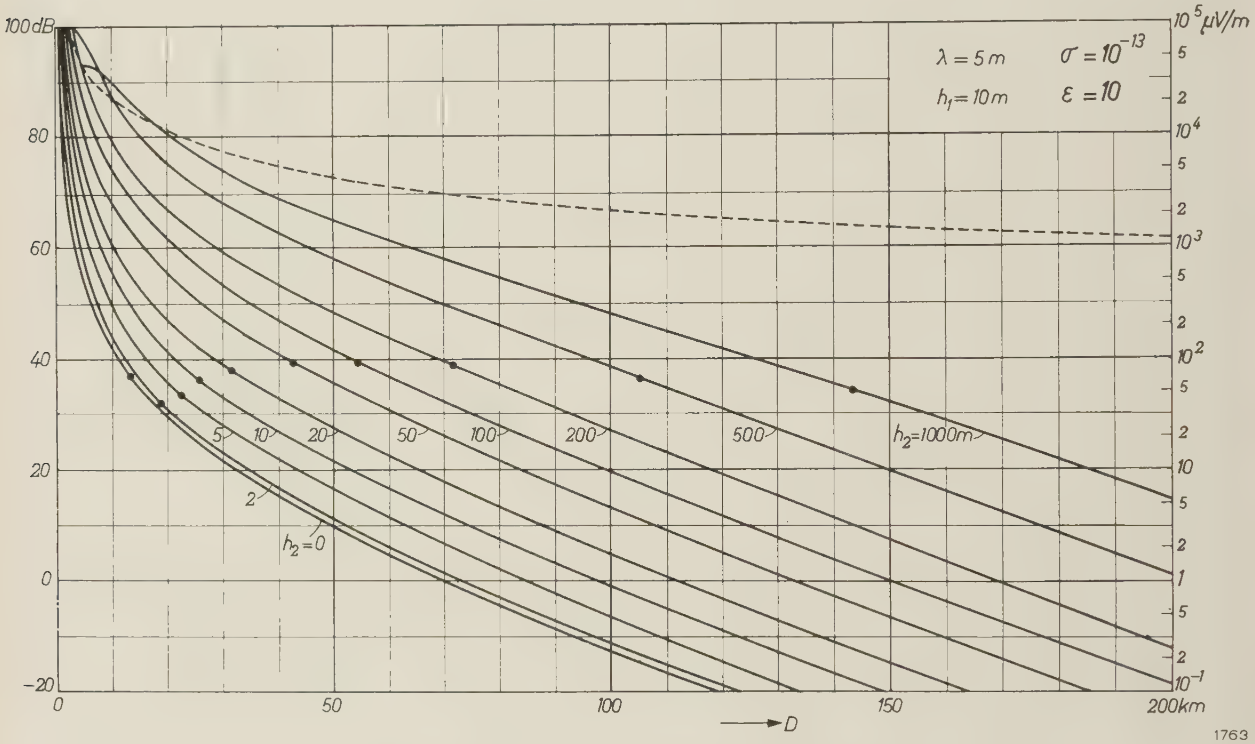

Fig. 2. Vertical component of the field strength of the ground wave (in dB and in µV/m) of a transmitter at height $h_1 = 10$ m, computed for various distances D and receiver heights h_2, for $\lambda = 5$ m and over slightly moist ground. The dashed curve would be obtained in the absence of the earth. The point on each curve indicates the horizon distance when the line connecting the transmitter and receiver touches the earth.

Non-linear circuits: relaxation oscillations

We shall now consider another field of research in which Van der Pol was particularly active, that of non-linear oscillations. It was understandable that in the 'twenties his attention should be drawn to this subject, since the advances already made in the application of triode circuits made necessary a deeper understanding of oscillation phenomena in order to round off the theory and to discern further possibilities. The simple linear theory of oscillatory effects is arrived at when the i_a-v_g characteristic of the triode (anode current versus grid voltage) is approximated by a straight line. The unsatisfactory point is that the amplitude of the excited oscillations then remains undetermined. The limitation of the amplitude is due to the decrease of the slope of the characteristic at either side of the operating point.

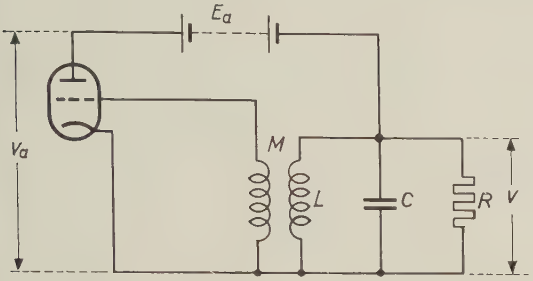

Fig. 3. Circuit for investigating oscillations of a triode circuit, in particular for studying the limitation of the amplitude (leading to Van der Pol's differential equation).

Fig. 3 shows a representative circuit for investigating these effects, in which M is the mutual inductance between the grid circuit and the LC circuit (with shunt resistance R) in the anode lead. We take the triode characteristic to be given by:

$$i_a = \Phi(v_a + gv_g), \quad \ldots\ldots \quad (1)$$

where g is the amplification factor of the triode. The non-linear function Φ of the variable $u = v_a + gv_g$ can be represented by a Taylor series in the neighbourhood of the point $u = E_a$:

$$\Phi(u) = \Phi(E_a) + \alpha\left(\frac{u - E_a}{k}\right) + \\ + \beta\left(\frac{u - E_a}{k}\right)^2 - \gamma\left(\frac{u - E_a}{k}\right)^3 + \ldots . \quad . \; . \quad (2)$$

The coefficients are normalized by the introduction of the parameter $k = g(M/L) - 1$. This circuit leads to the following differential equation for the time variation of the voltage v (see figure):

$$C\frac{d^2v}{dt^2} + \left\{\left(\frac{1}{R} - \alpha\right) + 2\beta v + 3\gamma v^2 + \ldots\right\}\frac{dv}{dt} + \\ + \frac{1}{L}v = 0. \quad . \; . \quad (3)$$

Van der Pol showed in the first place that the coefficient β, which determines the first non-linear term of the characteristic, so important for *detection*, does not in itself establish a finite value of the amplitude. The effect of this term will merely be to cause the angular frequency of the LC circuit to differ slightly from its value $(LC)^{-\frac{1}{2}}$ in the linear theory. The term with the coefficient γ is the first term to influence the amplitude. The following terms determine less essential properties. For example the fifth-order term in (2) is necessary in order to understand certain hysteresis effects, also studied by Van der Pol 9). In his amplitude investigations he therefore confined himself mainly to study the influence of the γ term, taking $\beta = 0$. By neglecting all higher terms in the expansion of the characteristic, eq. (2), he obtained from (3) a differential equation which, after introducing reduced variables both for the voltage v and the time t, assumed the form:

$$\frac{d^2y}{dx^2} - \varepsilon(1 - y^2)\frac{dy}{dx} + y = 0, \quad . \; . \; . \quad (4)$$

where $\varepsilon = \sqrt{L/C}\,(\alpha - 1/R)$.

This differential equation is known as "Van der Pol's equation". Initial investigation of this equation for *small* values of the dimensionless parameter ε showed that the solution, after a preliminary "time" x_0 of the order of $1/\varepsilon$, approximates to a normal sinusoidal oscillation of amplitude equal to 2. Van der Pol wondered what the corresponding solution would look like for larger values of ε, at least of the order of unity. The mathematical difficulties involved led him to apply a graphical procedure, the method of isoclines 10). In the case under consideration this amounts to reducing equation (4) with the aid of the new variable $dy/dx = z$ to the following first-order differential equation:

$$\frac{dz}{dy} = \varepsilon(1 - y^2) - \frac{y}{z}. \quad . \; . \; . \; . \quad (5)$$

Plotting z against y, this equation determines at every point a slope dz/dy, which can be indicated by a short dash. Drawing a continuous curve through a series of these dashes produces a curve which represents a possible solution in the variables y and z. A start can be made at various points, and in this way solutions constructed that satisfy the relevant initial conditions. Further numerical integration produces from each curve a solution for y as a function of x.

This procedure is illustrated in *fig. 4*, where various solutions are given for the relation between y and dy/dx, when $\varepsilon = 1$. *Fig. 5* shows the solution for y itself, corresponding to one of these, and also solutions for y for the cases $\varepsilon = 0.1$ and $\varepsilon = 10$, similarly

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9) B. van der Pol, Phil. Mag. **43**, 700, 1922.
10) B. van der Pol, Phil. Mag. **2**, 978, 1926.

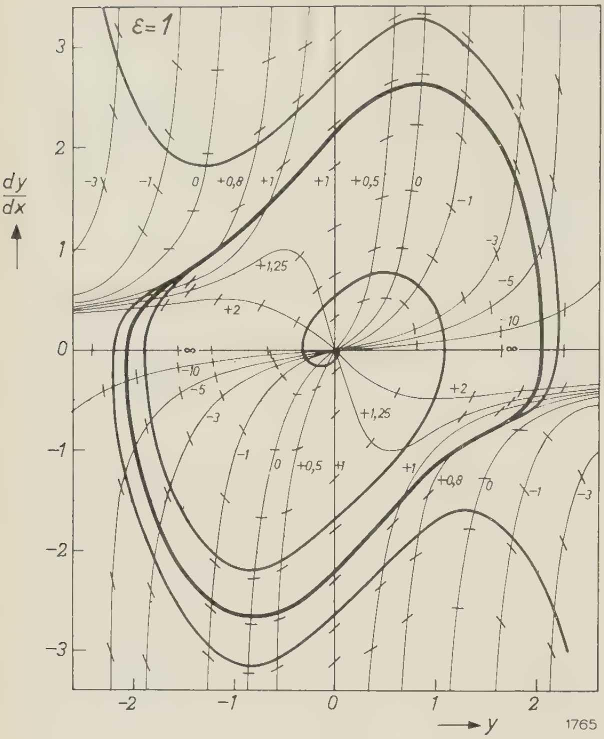

Fig. 4. Illustrating the isocline method of solving the differential equation (5), for $\varepsilon = 1$.

obtained. It follows from fig. 5 that, as the time increases, the solution approaches asymptotically to a non-sinusoidal periodic function, which differs more from a sinusoidal oscillation the larger ε is; the larger the value of ε the sooner is the asymptotic end state reached. Furthermore, the period in this end state for large values of ε is seen to approach a value Δx which is of the order of ε; it thus differs appreciably from the corresponding period for very small values of ε, which, in terms of the reduced time unit x, approaches 2π. Returning to the original variables v and t in eq. (3), this means that the period for large values of ε is of the order of magnitude of $L(a - 1/R)$. This is a *relaxation time*, i.e. a time that determines the decay of an aperiodic process. (Better known is the case occurring in other circuits where a relaxation time is determined by an RC product, see e.g. fig. 6.) For this reason, Van der Pol gave the name *relaxation oscillations* to these oscillatory effects which, at large values of ε, constitute the end state of processes defined by the differential equation (4).

Van der Pol first described these oscillations in 1926, after which he continued to investigate them, theoretically and experimentally, in cooperation with J. van der Mark [11]). A particular study was made of *forced* oscillations [12]), produced when an external alternating voltage is applied to a system like that in fig. 3. In the simplest cases the situation can be described by substituting for the right-hand side of Van der Pol's equation a term proportional to $\cos \omega t$. For small values of ε this provided a better understanding of already known properties of oscillator circuits. This applied particularly to the suppression of the free oscillation of a system that occurs when its frequency is close to that of the imposed external oscillation. The resultant synchronism with the latter oscillation was found to be particularly pronounced when, with increasing ε, the first, approximately sinusoidal, free oscillation gradually changes into a free relaxation oscillation. Another way of putting this is that the relaxation oscillations very readily assume the frequency of a superposed alternating voltage if the latter does

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[11]) B. van der Pol and J. van der Mark, Onde électrique **6**, 461, 1927.

[12]) B. van der Pol, Phil. Mag. **3**, 65, 1927.

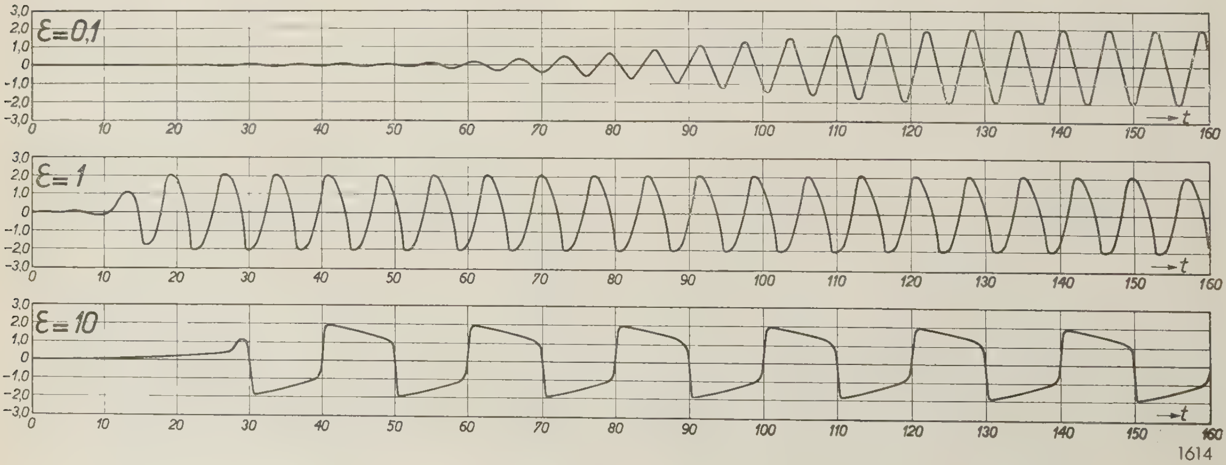

Fig. 5. Solutions of the differential equation (4), obtained by integration from solutions of eq. (5), for $\varepsilon = 0.1$, $\varepsilon = 1$ and $\varepsilon = 10$. (The variable along the abscissa, wrongly denoted by t, is the reduced quantity x.)

not differ too much from the frequency, determined by a relaxation time, of the natural free relaxation oscillations. If the circuit be modified so as to reduce this relaxation time to about *half* its original value, fairly abrupt synchronization will occur with half the frequency of the external oscillation, and after a further change with a *third* of this frequency, and so on. In other words, it is possible to generate relaxation oscillations that can successively be synchronized with an external oscillation and its sub-harmonics. These synchronization effects occur the more readily the larger the value of ε, or more generally, the more numerically important are the higher terms in eq. (2). Whilst the frequency of a relaxation oscillation can thus very easily be influenced, the amplitude is not nearly so readily controlled.

This synchronization with sub-harmonics later proved to be of great practical importance in the early development of television [13]). A relaxation oscillation is used in television for the scanning of successive lines of the picture. With suitable circuitry a succession of sub-harmonics can be formed, finally producing a relaxation oscillation with a synchronized frequency of 25 (or 30) c/s. This is then used to effect the return to the first line after the complete scanning of each frame. A simple circuit for demonstrating the frequency division of relaxation oscillations is shown in *fig. 6*, where the battery voltage E must be higher than the ignition potential V_1 of the discharge tube B. In the absence of the alternating voltage $E_0 \sin 2\pi f_0 t$, the capacitor C charges up via the high resistance R until its voltage V reaches V_1; it then discharges through the low resistance r of the ignited gas discharge. The discharge is extinguished when the capacitor voltage has dropped to a value V_2, after which the charging and discharging process is repeated. Applying now the alternating voltage $E_0 \sin 2\pi f_0 t$, and continuously reducing the frequency of the free relaxation oscillations (which is proportional to $1/RC$) by gradually increasing the capacitance C, it is easy to demonstrate the sudden occurrence of synchronization with the successive subharmonics having frequencies $f_0/2$, $f_0/3$, $f_0/4$ and so on. This is further illustrated in *fig. 7*, where the variable capacitance C is set out along the abscissa. The stepwise changes in the period of the forced oscillation can be read from the ordinate.

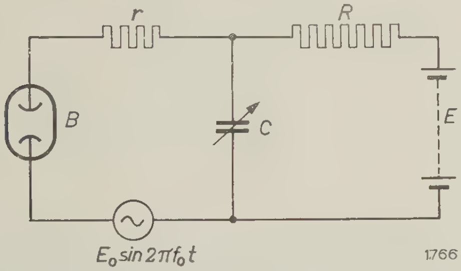

Fig. 6. Circuit for demonstrating the frequency division produced by the synchronization of relaxation oscillations with the sub-harmonics of an imposed voltage $E_0 \sin 2\pi f_0 t$.

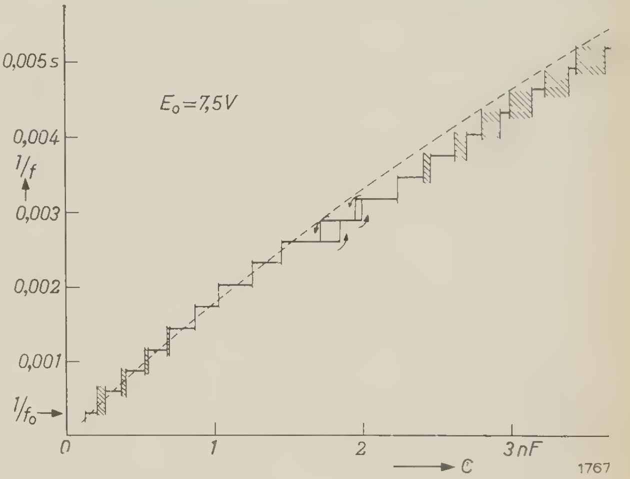

Fig. 7. Synchronization with sub-harmonics. The graph represents the period $1/f$ of the forced relaxation oscillation of the circuit in fig. 6 excited at a constant frequency f_0 while the capacitance C is gradually varied.

Subsequently, in collaboration with C. C. J. Addink, Van der Pol entered upon the study of more general synchronization phenomena. It proved possible to obtain synchronization not only with frequencies $f_0/2$, $f_0/3$, $f_0/4$ etc., but also with frequencies $(n/m)f_0$, where n and m are arbitrary integers (n may even be $> m$). For these experiments the frequency of the imposed oscillation was varied instead of one of the parameters of the relaxation oscillator [14]).

Characteristic of the operation of the circuit in fig. 6 is the occurrence of an essentially aperiodic process (in this case the exponential charge and discharge of the capacitor) which is periodically interrupted and restarted at certain critical values of a relevant parameter. Such situations are frequently found in nature, and they always give rise to relaxation oscillations. Van der Pol was struck by their very number and variety. He drew up a long list of examples, which included: the aeolian harp, the scratching of a knife on a plate, the periodic recurrence of epidemics and economic crises, the periodic density variation of two (or another even number) types of animals that live together and of which one serves as food for the other, the sleep of flowers, phenomena associated with periodic rain showers after the passing of a meteorological depression, and finally the beat of the heart.

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[13]) J. van der Mark, An experimental television transmitter and receiver, Philips tech. Rev. 1, 16-21, 1936.

[14]) B. van der Pol, Actualités scientifiques et industrielles No. 718, published by Hermann, Paris 1938, p. 69-80.

The latter example led Van der Pol to a very interesting and instructive practical application of relaxation oscillations, namely an electrical model for simulating all the rhythmic movements of the human heart [15]). With this system it was a simple matter to record "electrocardiograms", and to study the normal heart beat as well as cardiac disorders. The importance of the system to medical science will be obvious. The heart may be regarded as consisting of three systems: the sinus venosus, the two auricles and the two ventricles. The operation of each system can be simulated by a circuit of the type in fig. 6. The unilateral effect of the sinus on the auricles, and of the auricles on the ventricles, was represented by non-amplifying triodes. After other details of the operation of the heart had been taken into account the model illustrated in *figs. 8* and *9* was finally produced. The ignition of the gas discharges in the circuits simulating the auricles and ventricles represents respectively an auricular and a ventricular systole. The synchronizations in this model with subharmonics of the resultant dominant heart beat can be recognized in the human heart in certain cases.

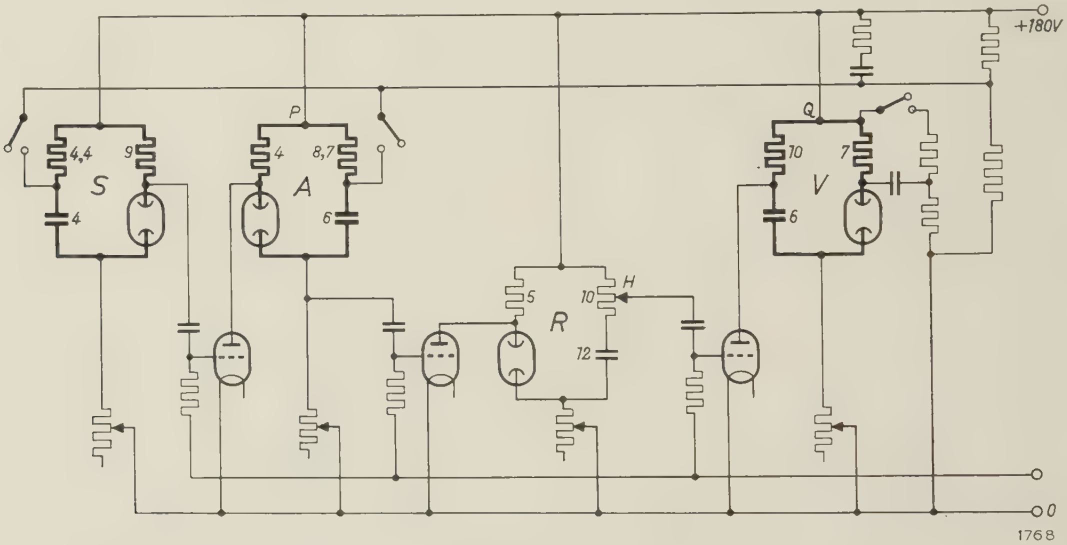

Fig. 8. Circuit simulating the functioning of the heart. Resistance values in kΩ, capacitances in μF. The three circuits *S*, *A* and *V*, each of which can enter into relaxation oscillations and which are coupled in a special way, represent respectively the sinus venosus, the auricles and the ventricles. *R* is a delay system which simulates the transit time of a stimulus through the bundle of His. The ignition of each of the neon lamps in *S*, *A* and *V* corresponds to a systole of that part of the heart. The coupling between auricles and ventricles can be varied with the potentiometer *H* (Erlanger's experiment). Electrocardiograms can be made by recording the current in one of the leads from point *P* or *Q*.

The development of the heart model out of the work on relaxation oscillators was a side-track similar to that which led from considerations of radio-wave propagation to a new wave-theoretical treatment of the rainbow. Van der Pol liked to point out how radio problems were apt to draw attention to fields which, at first sight, seem to have very little connection with radio [16]).

Transient phenomena and operational calculus

Operational calculus was introduced by Heaviside with the original object of providing a simple means

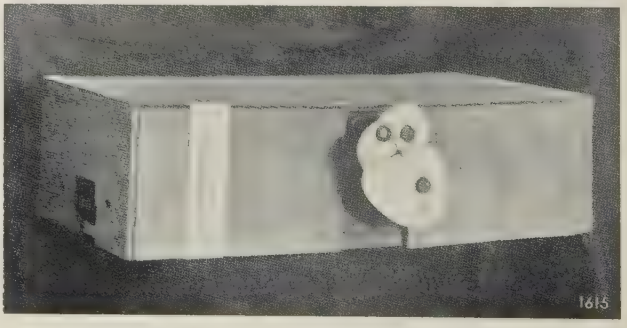

Fig. 9. Photograph of the heart simulator. The three neon lamps of the circuits *S*, *A* and *V* (see fig. 8) are mounted on the three corresponding parts of the anatomical representation of the heart.

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[15]) B. van der Pol and J. van der Mark, Phil. Mag. **6**, 763, 1928.

[16]) The examples mentioned and various others will be found in: B. van der Pol, Proc. World Radio Convention, Sydney 1938.

of studying switching transients in electrical networks. The methods devised were later developed into a system of calculus which has proved to be extremely useful in mathematics as a whole. No one saw this more clearly than Van der Pol, who demonstrated by numerous examples the possible applications of operational methods to widely diverse fields of study.

To illustrate the starting point of operational calculus we shall briefly discuss an aspect of the historical development of alternating current circuit theory, often examined by Van der Pol. At first the aim was to determine how a certain system responds to an externally applied harmonic vibration $\cos \omega t$, or, more generally, $\exp j\omega t$, where $\cos \omega t$ is the real component. Investigations were made to ascertain in what way the properties of the system depend on the frequency $f = \omega/2\pi$. As a case in point we shall consider an amplifier. The ratio $G(j\omega)$ between the output voltage and input voltage were studied in the case where both were proportional to $\exp j\omega t$. If the function $G(j\omega)$ is known, we can calculate with the aid of Fourier analysis the output voltage for any arbitrary time function of the input voltage. The case of the response to a discontinuous input voltage can also be treated in this way. In applying this procedure it was often overlooked, however, that the complex function $G(j\omega)$ depends on *two* real (though mutually related) functions, and that it is consequently not enough to take account only, for example, of the modulus $|G(j\omega)|$. This was discussed some considerable time ago in this journal by J. Haantjes [17]). On the other hand, all properties can equally be derived from the knowledge of a single real function, such as the unit step function response $G_s(t)$. This is defined as the output voltage resulting from the application at a given moment $t = 0$ of a direct voltage of unit magnitude to the input side, when no voltage was present for $t < 0$. (By giving the input function *unit* voltage, G_s is a dimensionless function of time.) All characteristics of the amplifier can be described in terms of this unit function. It can be shown that for a given arbitrary primary voltage $v_{pr}(t)$ at the input, the output voltage of the amplifier will be

$$v(t) = \int_{-\infty}^{+\infty} G_s(t-\tau)\,\frac{d}{d\tau}\,v_{pr}(\tau)\,d\tau = \int_{-\infty}^{+\infty} G_s(\tau)\,\frac{d}{dt}\,v_{pr}\,(t-\tau)d\tau. \quad . \quad . \quad (6)$$

This representation is simpler than that where the system is described in terms of the above-mentioned complex function $G(j\omega)$, which defines the behaviour of the system in the case of a continuous, harmonic input voltage. If the latter function is used, the effect of an arbitrary input voltage $v_{pr}(t)$ is given by the expression:

$$v(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} G(j\omega)\left[\int_{-\infty}^{+\infty} v_{pr}(\tau)\,e^{j\omega(t-\tau)}\,d\tau\right] d\omega, \quad (7)$$

which is generally more difficult to handle for a non-periodic $v_{pr}(t)$ than eq. (6).

The above shows that it is sometimes simpler to study the behaviour of a system with reference to a discontinuous process (here the sudden application of a unit step voltage, giving the step-function response $G_s(t)$), than with reference to a continuous process (the continuous application of an alternating voltage $\exp j\omega t$, from $t = -\infty$ to $t = +\infty$). An objection to working with the unit-step function might be that, although mathematically easier to handle in such cases, true discontinuity cannot in reality be achieved. It should be remembered, however, that any discontinuous process can be regarded as the limit of a physically realizable process, and that, strictly speaking, there are in fact no perfectly continuous processes either. Van der Pol once remarked that there seemed to have been a long-standing aversion to the study of discontinuous phenomena. He described this aversion wittily as a "horror discontinuitatis", resembling the "horror vacui" assumed by 17th century physicists.

We have discussed these preliminaries to show how useful Heaviside's operational methods can be for the study of electrical systems. This appears from the fact that eq. (6) — a so-called composition product or convolution integral, in this case of the two functions $G_s(t)$ and $v_{pr}'(t)$ — is particularly suitable for treatment by the methods of operational calculus. In the version of operational calculus adopted by Van der Pol and the author [18]), to any arbitrary function $h(t)$ (the "original") one may allot a new function, the operational "image" $f(p)$, defined by the following Laplace integral:

$$f(p) = p\int_{-\infty}^{+\infty} e^{-pt}\,h(t)\,dt\,. \quad . \quad . \quad . \quad . \quad (8)$$

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[17]) J. Haantjes, Philips tech. Rev. **6**, 193, 1941.

[18]) B. van der Pol and H. Bremmer, Operational calculus, based on the two-sided Laplace integral, Cambridge Univ. Press 1950. — The version of the operational calculus developed in this book is characterized by the fact that the integral in eq. (8) has the lower limit $-\infty$, as opposed to the more conventional definition used by Carson, Doetsch, Wagner *et al.*, where the lower limit is always 0. The advantages of using $-\infty$ as the lower limit are explained in the book.

The image of a convolution integral is simply the product of the images of the individual factors. In the case of eq. (6) we therefore find the operational image of the function $v(t)$ as the product of the images of $G_s(t)$ and $v_{pr}'(t)$. By finding the original of this operational image we then have a complete solution of the problem.

The other expression for $v(t)$, eq. (7), which must obviously be equivalent to eq. (6), has a form which does not lend itself so well to an operational solution. Since eq. (7) is precisely the form most suited for the treatment of periodic processes, we can see why the operational calculus is less useful for dealing with such processes than with transients. Nevertheless, eq. (7) must evidently also have some relation to operational calculus; this relation becomes clear if we recall that (7) represents a Fourier integral which, after appropriate substitutions, can be put in the form of the Laplace integral (8). It is precisely the linking of operational methods to an exponential integral of this kind (which is related to the familiar Fourier integrals) that has made this calculus into a rigourously grounded mathematical tool.

In order to apply operational methods to the solution of all kinds of concrete problems, such as determining the above-mentioned integral (6) with given explicit functions, it is evidently useful to have a list of images available that correspond to originals given by elementary functions. Van der Pol called such a list his "dictionary", whilst the rules of operational calculus were the "grammar". One of these rules, for example, is the foregoing theorem concerning the convolution integral.

Armed with such a grammar and dictionary Van der Pol studied numerous properties of electrical networks, and of filters in particular. He arrived at theorems, often of very general application, which concerned for instance the relation between the unit function response of a given filter and that of a new filter derived from it; the latter was produced from the original by replacing all inductances by capacitances, and vice versa. General theorems of this kind sometimes led to interesting special cases. One notable example was the discovery that the charging of a system of capacitors from a D.C. source always takes place with an efficiency of 50%, irrespective of the number of resistances and inductances in the circuit [19]). This is to say that the final energy of the charged capacitors amounts to half the energy which the source, after being suddenly switched on, must deliver in order to sustain the currents which must flow until the system reaches its final state.

Many of the mathematical relations discovered by Van der Pol were arrived at quickly by the skilful manipulation of the methods of operational calculus. An example of such a relation in the case of continuous functions is the remarkable and previously unknown identity;

$$a\int_0^\infty \mathrm{e}^{-at}\sin^{2n}t\,\mathrm{d}t = \frac{(2n)!}{(a^2+2^2)\,(a^2+4^2)\,\ldots\,(a^2+4n^2)}\,, \quad \text{for} \quad \mathrm{Re}\;a > 0. \quad \ldots \quad (9)$$

Van der Pol liked in particular to work on relations for *discontinuous* functions. A representative example is the function $A_2(t)$, which indicates the number of pairs of integers (m,n) for which $m^2+n^2<t$; operational methods led at once to the surprising relation:

$$\tfrac{1}{4}\{A_2(t)-1\} = \left[\frac{t}{1}\right] - \left[\frac{t}{3}\right] + \left[\frac{t}{5}\right] - \left[\frac{t}{7}\right] + \ldots, \quad (10)$$

where the symbol [ ] here represents the largest integer $\leqslant$ the number within the brackets.

As a last typical example of the application of operational calculus we mention the "moving average" or "sliding mean" theory, which was extensively studied by Van der Pol. Starting from an arbitrary function $g(t)$, the sliding mean is defined by the new function

$$g^*(t) = \int_{t-\Delta}^{t+\Delta} g(\tau)\,\mathrm{d}\tau. \quad \ldots \quad (11)$$

In many cases we want to know the function $g(t)$ when $g^*(t)$ has been measured. Take, for example, the intensity $g(\omega)$ as a function of frequency ω in a spectrogram recorded with a spectrograph. Owing to the necessarily finite width of the slit in the spectrograph, the actual intensity $g(\omega)$ is smeared out into a generally less-rapidly varying function $g^*(\omega)$. The operational solution of this problem leads to many ways in which g can be expressed by a series in g^*, suitable for numerical use.

Other subjects

In the foregoing we have discussed the most important general subjects on which Van der Pol was engaged. We shall now touch briefly on various other, more specialized problems on which he worked for shorter or longer periods — on some all his life —

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[19]) B. van der Pol, Physica 4, 585, 1937.

and which will be found in the bibliography of his numerous publications [20]).

Amongst the earliest researches of Van der Pol were special subjects concerned with the theory of electrical networks, including current distributions in an arbitrary number of coupled circuits. His interest in antenna theory was first aroused by investigations into the radiation and natural frequency of antennae possessing end capacitance, during which the inadequacy of the concept of radiation resistance was discussed. In England he carried out experimental research on the conductivity of seawater, in connection with its bearing on the propagation of radio waves. When he joined the Philips laboratory his interest extended to virtually all problems of radio technology, then still in its early stages. Partly in cooperation with K. Posthumus, Y. B. F. J. Groeneveld, T. J. Weijers, G. de Vries, C. J. Bakker, W. Nijenhuis, C. J. Bouwkamp and others (mentioned elsewhere in this article) he investigated the general properties of triode characteristics, the distribution of the electrical field and electron paths inside a triode, the theory of grid detection, general properties of oscillators and filters (including the equivalent circuit of a quartz oscillator, and the non-linear theory of hysteresis effects in two coupled circuits, one of which is part of a triode generator), noise in radio valves, radiation patterns of beam antennae, and many other subjects.

Van der Pol had an important share in the preparatory work leading to the introduction of radio broadcasting in the Netherlands. Together with R. Veldhuyzen, M. Ziegler and J. J. Zaalberg van Zelst he did extensive research into the field-strength distribution of broadcast waves over the Netherlands. The results were plotted on charts, showing contours of constant field strength. The deviation of these contours from a circular shape indicated the influence of the type of ground traversed between transmitter and receiver. In the chart for the Hilversum transmitter on 298.8 m wavelength, for example, the absorbent effect of the city of Amsterdam and of the dry sandy ground of the chain of hills east of Utrecht could be clearly seen. This chart led Van der Pol to point out that the field-strength distribution in the Northern provinces would change after the damming of the Zuiderzee, since the conductivity of this large expanse of water would decrease as it became progressively less salty. This was confirmed by later measurements.

When a new transmitter was planned to replace the old one at Hilversum, the reciprocity theorem was applied to determine its most favourable location: instead of starting with a transmitter in a central (though not yet determined) point, measurements were made of the fields due to two auxiliary transmitters in two distant points in the country, where reception from a centrally-situated transmitter would have been weakest. The most favourable site for the new transmitter was found by determining, in the central region of the country, at what point the received field was strongest on the line connecting the points of intersection of corresponding contours charted for both auxiliary transmitters (*fig. 10a*, *b* and *fig. 11*).

Long before there was any question of the practical exploitation of frequency modulation, which has since become so important in broadcasting, Van der Pol had investigated the theory of this system of modulation. Fundamental insight here requires much more mathematical knowledge than the simpler system of amplitude modulation, since the differential equations involved can no longer be solved by time functions proportional to exp $j\omega t$. In the first place, Van der Pol gave a suitable definition for the vague concept "instantaneous frequency", ω_{inst}. Writing the general frequency-modulated signal as:

$$A \cos \varphi(t) = A \cos \left[\omega_0 t + m\omega_0 \int_0^t g(t)\mathrm{d}t + \Phi\right], \quad (12)$$

this definition reads:

$$\omega_{\text{inst}}(t) = \frac{\mathrm{d}}{\mathrm{d}t}\,\varphi(t) = \omega_0 \,\{1 + m\,g(t)\}. \quad . \quad (13)$$

Together with F. L. H. M. Stumpers he then investigated the so-called quasi-stationary approximation whereby the currents and voltages assume values at any given instant that conform to the then existing value of the instantaneous frequency as defined by (13). It was found that the current produced in an impedance as a result of a frequency-modulated voltage is then solely determined by the phase characteristic of the impedance [21]). In the 'forties Van der Pol also carried out experimental research on frequency modulation, but lack of space must preclude its discussion here.

In the theoretical investigation of frequency modulation an important part was played by

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[20]) A complete bibliography of Van der Pol's publications, compiled by C. J. Bouwkamp, will appear shortly in Philips Res. Repts.

[21]) B. van der Pol, J. Instn. Electr. Engrs. **93 III**, 153, 1946.

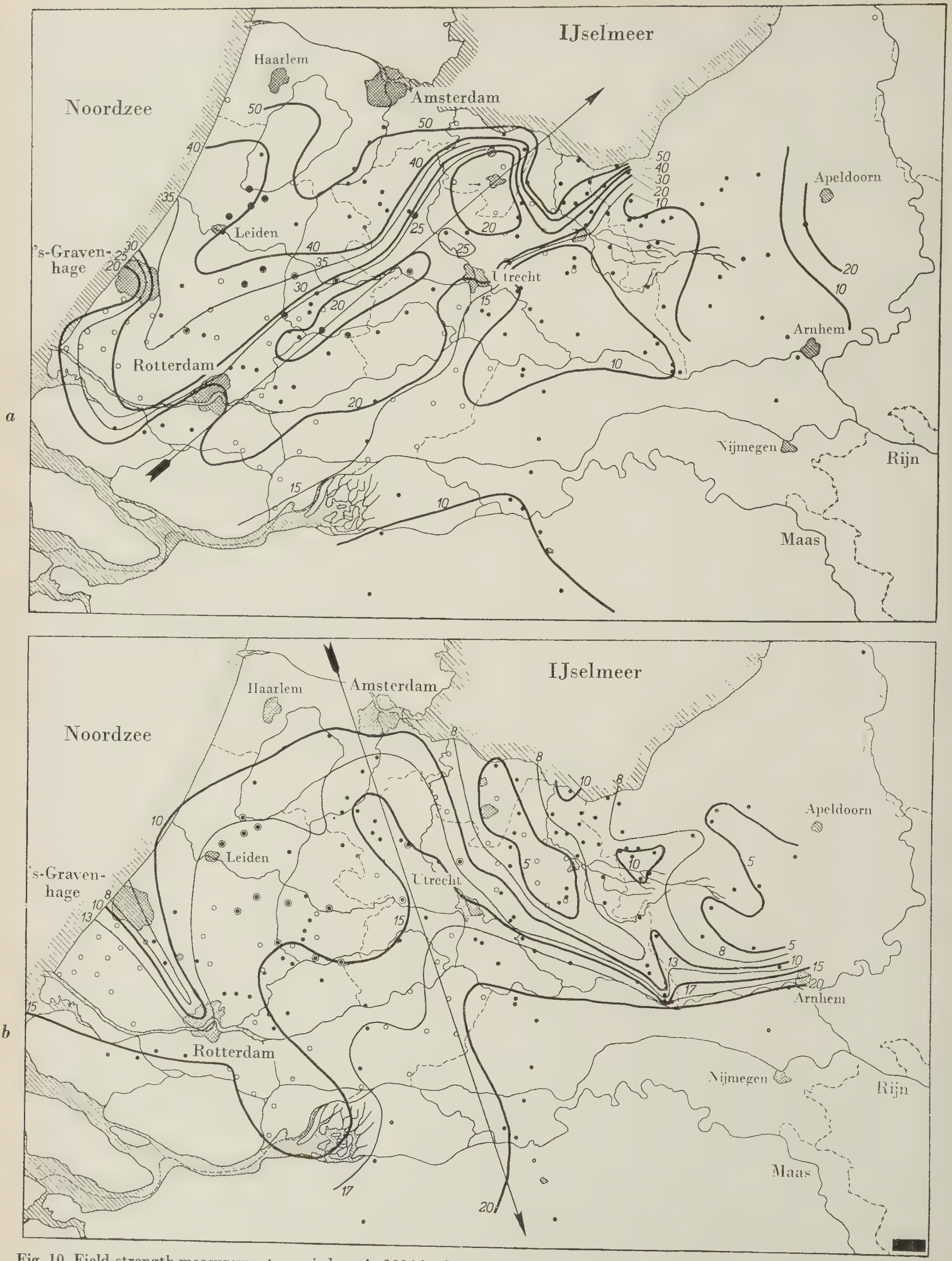

Fig. 10. Field-strength measurements carried out in 1934 in the central region of the Netherlands for determining the most favourable site for the new broadcasting station, planned at that time. In two remote parts of the country, where the poorest reception might be expected from a central transmitter, two identical auxiliary transmitters were erected, one at the Dollard estuary (wavelength 325 m), and the other at Maastricht (wavelength 317 m).
a) The field distribution of the Dollard transmitter represented by contours of constant field strength. The long arrow points to the transmitter.
b) The same for the Maastricht transmitter. (Note the shadow effect due to Rotterdam.)

Mathieu's differential equation. More generally, Van der Pol applied himself to the study of differential and difference equations related to radio problems. These studies again led to many and various investigations that had a less direct connection with physical or technical questions, or were even of a purely methematical nature. They included theta functions, elliptic functions, the gamma function, the theory of numbers, the properties of prime numbers, and so on. Nevertheless, these problems cannot be seen as entirely distinct from radio technology. Theta functions, for example, are related to the theory of potential functions, and Van der Pol's interest in the theory of numbers was inspired by the part it plays, for instance, in the above-mentioned synchronization effects in relaxation oscillators. Incidentally, these abstract researches also led to a quite unexpected "industrial by-product", as a result of Van der Pol's study of the distribution of Gaussian prime numbers. In the system of all complex numbers with integral real and imaginary parts, these are the numbers that cannot be obtained as the product of two of them. When all these two-dimensional prime numbers are drawn in the complex plane, a curious pattern is produced (*fig. 12*). This was used as a tablecloth pattern which had particular success in America.

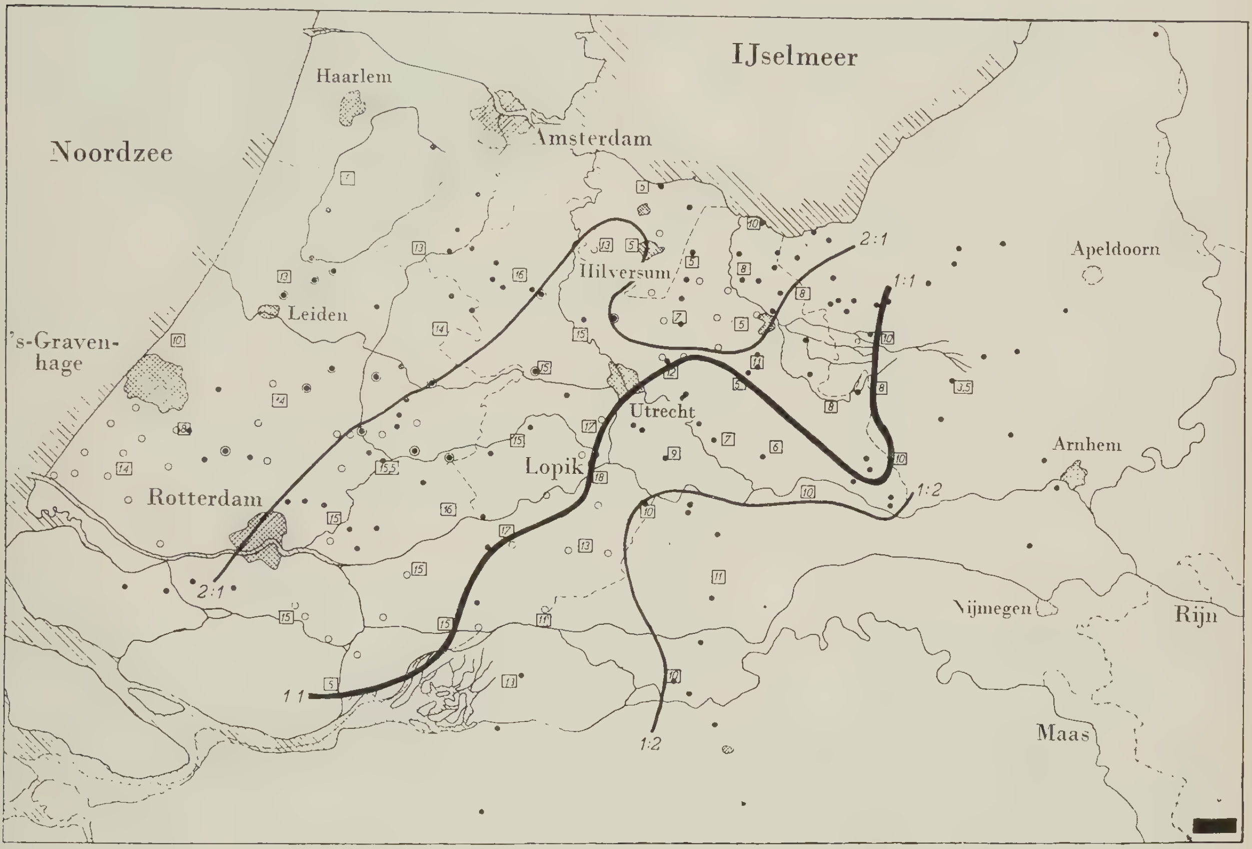

Fig. 11. The bold lines connect the places where the two auxiliary transmitters are received with equal strength (thick central line, 1 : 1) or with an intensity ratio of 2 : 1 and 1 : 2. As can be seen, the highest field strength is found on the thick line at the point 18 near the village of Lopik (where in fact the station was subsequently built). The numbers at other places indicate the relative value of the field which, if the transmitter of the station had been sited there, would have been obtained in the more unfavourable of the two poor-reception areas (Dollard or Maastricht). (Fig. 10*a*, *b* and fig. 11 are taken from B. van der Pol, Rapport van de veldmetingen van twee bij de Dollard en bij Maastricht opgestelde proefzenders, T. Ned. Radiogen. **7**, 173-195, 1935.)

The association of mathematics and radio technology sometimes worked in the opposite direction. We refer to an investigation whereby Van der Pol, in cooperation with C. C. J. Addink, used electrical techniques for studying a mathematical function [22]). The investigation concerned the determination of the zero points of Riemann's ζ function, normally defined by the series:

$$\zeta(z) = \frac{1}{1^z} + \frac{1}{2^z} + \frac{1}{3^z} + \dots . \quad . \; . \; . \; . \qquad (14)$$

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[22]) B. van der Pol, Bull. Amer. Math. Soc. **53**, 976, 1947.

Although this series can be used only for Re $z > 1$, the function can nevertheless be defined for the whole complex plane by means of an analytic continuation. Riemann had conjectured that the function thus defined would possess an infinite number of zero points, all of which would lie on the line Re $z = \frac{1}{2}$. By means of a relation derived by Van der Pol, the functional values along this line can be associated with the simple function:

$$\mathrm{e}^{-x/2}\left[\mathrm{e}^{x}\right] - \mathrm{e}^{x/2} \quad . \; . \; . \; . \; . \; . \quad (15)$$

(the symbol [ ] has the same meaning as in eq. (10)). In fact, when the function (15) is cut off after a convenient large value of x, and then periodically continued, the coefficients of the Fourier series of the new function obtained in this way represent approximations to a series of values of the function $\zeta(z)/z$ along the line Re $z = \frac{1}{2}$. The problem of determining the zero points conjectured by Riemann was thus reduced to a Fourier analysis of the said function derived from (15). This Fourier analysis was carried out in an elegant experiment in which the sawtooth pattern of one period of the function in question was cut along the edge of a disc (*fig. 13*). The disc was rotated at a highly constant speed and light was passed through a radial slit and caused to fall on the serrated edge and on a photocell placed behind it. The photo-current thus obtained, varying with time, was mixed with a sinusoidal current of frequency f. The resultant signal, containing the difference tones between f and all harmonics of the periodic function derived from (15), was used to excite a mechanical resonator of very sharp resonance. When the frequency f was now slowly raised from zero, each successive harmonic of the above function excited the resonator in turn with its difference tone, and with an intensity corresponding to the relevant Fourier coefficient. By recording the excitation (as a function of f) a spectrum was obtained in which the required zero points of the ζ function could immediately be seen as minima; see *fig. 14*.

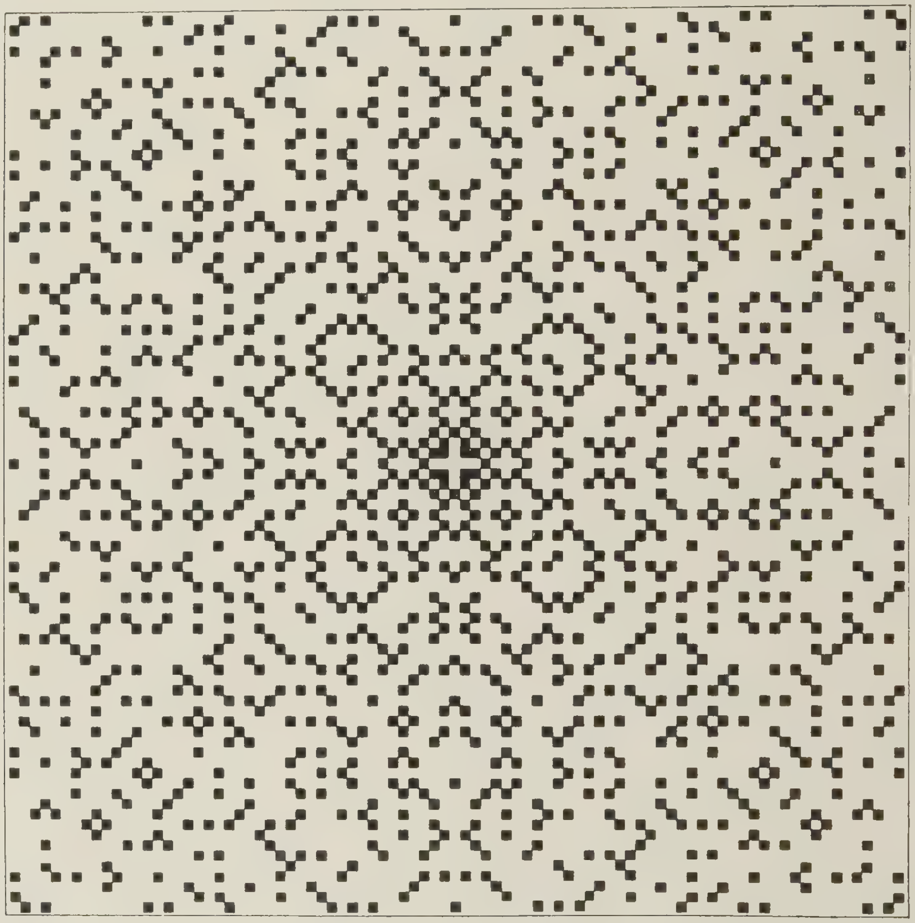

Fig. 12. The black squares represent in the complex plane (with origin in the centre) the complex prime numbers of Gauss, i.e. the numbers $(m + \mathrm{j}n)$ that cannot be written as $m + \mathrm{j}n = (a + \mathrm{j}b)(c + \mathrm{j}d)$, where m, n, a, b, c, d are integers. This pattern was woven into a fabric marketed by an Eindhoven textile factory.

Musical theory

We shall now turn from Van der Pol's scientific and technical investigations and in conclusion touch briefly on his interest in music. Although deeply interested in analysing the structure of a musical work, he realized that this could never be an approach to the creative aesthetic element —

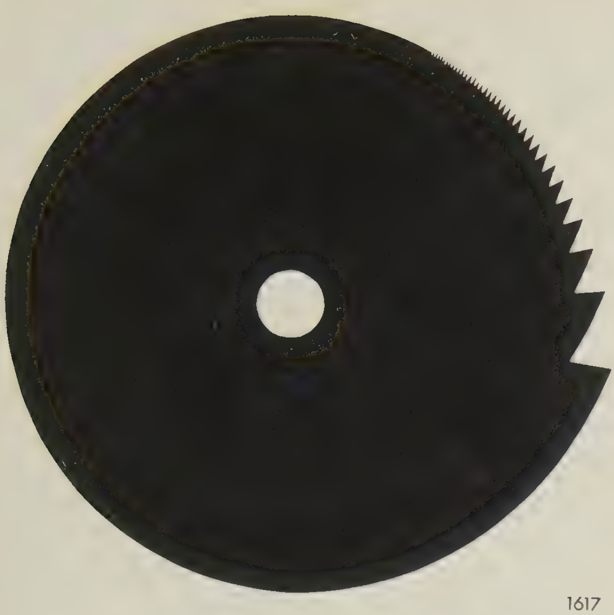

Fig. 13. Disc with edge serrations representing part of the function (15), for experimentally determining the zero points of Riemann's ζ function.

an element for which, as a competent musician himself, he had a deep understanding. It was his preoccupation with the theory of numbers that led him to look for mathematical laws in harmonic intervals[23], or, for example, to find numerical definitions for concepts such as dissonance. In particular he discovered that the fractions presented by the successive intervals in the diatonic scale could be contained in a Farey series. In this case it is the series of all irreducible fractions whose denominators and numerators never exceed the number 5. Only two tones from the diatonic sequence (the *b* and the *d*, with *c* as the fundamental) do not fit into the Farey series, and the remarkable thing is that it is precisely these tones about which there is most uncertainty in tuning (if one is not bound to the equal-temperament scale).

It will not be surprising after the foregoing that Van der Pol emphasised that the sub-harmonics of a given note can be produced with the aid of synchronization effects on relaxation oscillations. In this way it was possible to confirm experimentally a postulate put forward by the music theorist Hugo Riemann, namely that the minor triad can be regarded as the combination of the 4th, 5th and 6th sub-harmonics of a given tone, just as applies to the major triad in respect of the 4th, 5th and 6th upper harmonics of another tone.

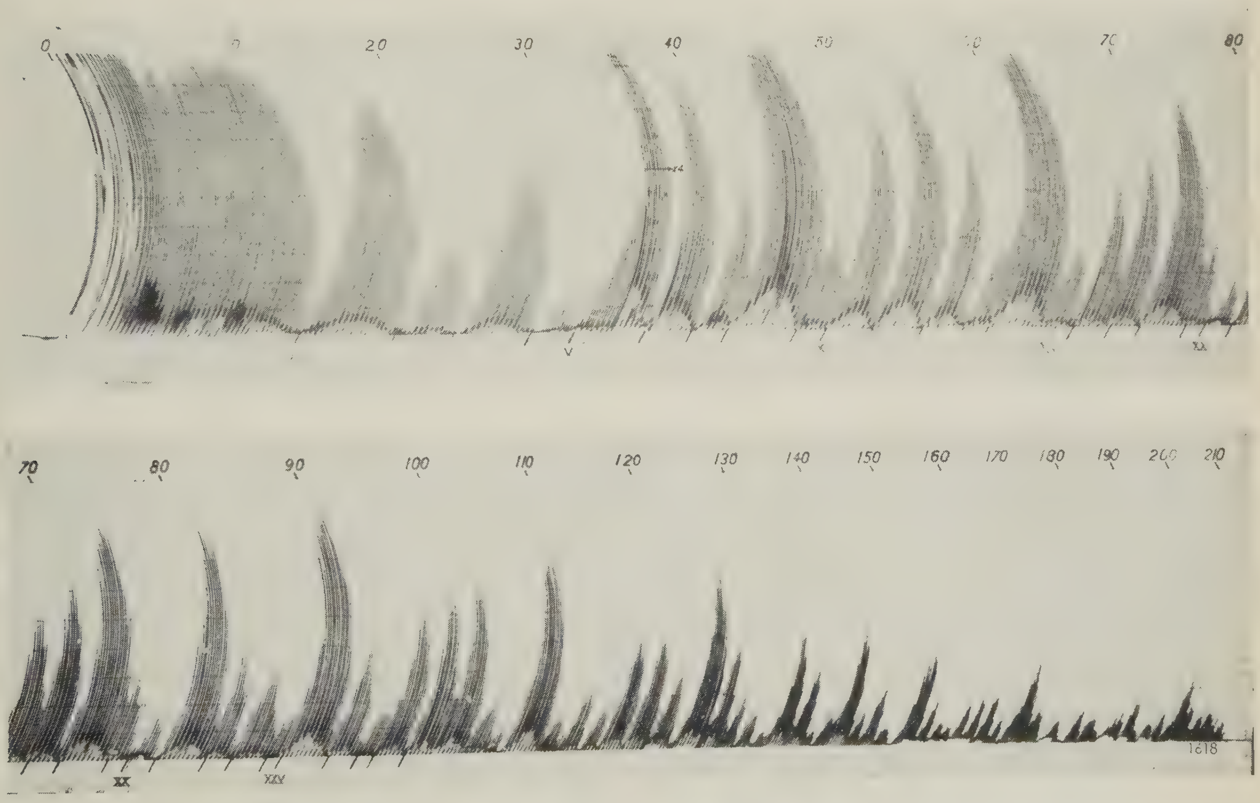

Fig. 14. Recording obtained with the aid of the disc in fig. 13, giving the Fourier coefficients of the function cut into the disc. The minima in the recording are the zero points of the ζ function on the line Re $z = \frac{1}{2}$. The first 29 zero points, denoted by dashes and Roman figures, were already known from calculations. (The imaginary part of z is set out along the top edge.)

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[23] B. van der Pol, Muziek en elementaire getallentheorie, Arch. Mus. Teyler **9**, 507-540, 1942 (in Dutch).

Van der Pol possessed absolute pitch, and was able, for example, to tune his violin to the right pitch without a tuning fork. He enjoyed exercising this gift and it was typical of the man that he should develop from it yet another interesting investigation. Together with Addink he devised an apparatus using an oscilloscope to check the tuning of an orchestra during a performance [24]). From hundreds of observations, both on permanently tuned instruments and on orchestras, they found that the middle *a* varied from 430 to 447 c/s, and also that there were quite distinct, more or less systematic variations of this frequency during a performance.

Van der Pol's enquiring mind led him to apply the principles of science to music. For him, however, there existed a much profounder relation between these fields. There is perhaps no more fitting way to end this review of the many-sided work of the great man of science that Van der Pol undoubtedly was, than to quote his own words, from a lecture given at the Teyler Foundation in Haarlem:

"Is there really any difference between the inspiration that leads to the conception of a beautifully flowing melody, a rich theme or a brilliant modulation, and that which leads to the conception of a new, elegant, unexpected mathematical relation or postulate? Are not both born in the same mysterious way and do they not both often demand laborious development? Art connotes skill, and skill too is the handmaid of science."

Summary. The review given deals principally with the work done by the late Van der Pol in the years from 1922 to 1949, when he was with the Philips Research Laboratories at Eindhoven. The main subjects discussed are the propagation of radio waves, especially the influence of the ground conditions on ground-wave propagation; non-linear circuit phenomena (in particular relaxation oscillations described by the "Van der Pol equation", and their synchronization); transients and operational calculus, which (following Heaviside, whom he greatly admired) Van der Pol used as a fruitful heuristic and later rigorously founded method. A brief discussion is devoted to various other subjects treated by Van der Pol in his numerous publications, including mathematical investigations prompted by problems of radio technology, such as the application of Mathieu's equation to frequency-modulation problems, and also studies relating to subjects of pure mathematics, such as elliptic functions and the theory of numbers (especially the properties of prime-numbers). Finally some examples are given of Van der Pol's interest in music and the theory of music.

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[24]) B. van der Pol and C. C. J. Addink, Philips tech. Rev. 4, 217, 1939.

DEMOUNTABLE SEALS FOR GLASS HIGH-VACUUM EQUIPMENT

by B. JONAS *) and G. SEITZ *).

666.1.037.4

An interesting variant is described of known methods of sealing glass components. The crux of the method is the inclusion of a metal ring in the seal. This makes it possible to disconnect the sealed parts in a matter of seconds, without causing damage.

There are many industrial products in which replacements or modifications have to be made after assembly. For this purpose it must be possible to dismantle the product without causing significant damage. In the case of products made partly of glass, this reversal of the fabrication process may present difficulties. For conventional electron tubes and gas-discharge tubes the vacuum envelope and the seals are therefore constructed without a view to dismantling. Nevertheless, glass vacuum vessels that can be dismantled are quite important in certain products that are expensive and made only in small quantities and in laboratory work (experimental apparatus with interchangeable components) and development work [1]). The ways and means that have been devised to circumvent these difficulties are many and various, and include ground-glass joints of diverse kinds, rubber gaskets and wax seals, etc. The drawback of such devices is that they do not allow degassing at a sufficiently high temperature; as a rule, therefore, they can be used only if the glass tubes to which they are fitted are kept continuously connected to a pump during operation.

We shall describe here a method based on a technique which has been used for certain radio tubes, and which has now been developed to meet more stringent requirements. First a few words about the older technique.

In order to be able to dismantle a vacuum vessel (of hard or soft glass) the glass envelope can be made in two parts that meet in a circular seam. If this seam is filled with a glaze which has a low melting point (*fig. 1*) the joint can later be opened again by melting the glaze — without exceeding the softening point of the glass envelope — in exactly the same way as two metal parts soldered together are disconnected by melting the soldered joint. As stated, a "glazing" technique of this kind was used for a time in the manufacture of radio tubes. In the "Rimlock" range of tubes [2]), for example, the glass parts were not fused directly together, but bonded by means of a glaze seal, as in fig. 1. This technique was used here to prevent damage to the cathode, not with a view to the possibility of dismantling the tube.

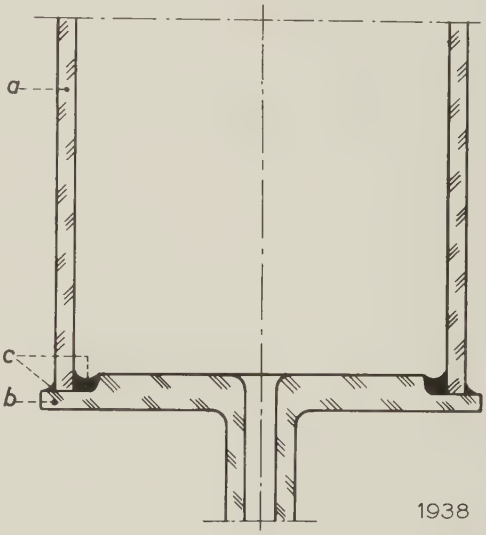

Fig. 1. Schematic representation of a glass seal using a glaze. *a* cylindrical glass envelope, *b* base, *c* melted glaze.

Having regard to the widely different properties of the types of glass used in tube manufacture (see *Table I*) the question arises whether such a method could find general application or would only be feasible in special cases. To effect a bond as described above, the following conditions must be fulfilled:

1) In the interval from room temperature to the temperature at which the tube glass begins to soften, the thermal expansion of the glaze must match that of the tube glass (to a first approximation they should be equal).
2) The flow point of the glaze must be sufficiently low so that softening and consequent deformation of the glass envelope cannot occur.

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*) Zentrallaboratorium Allgemeine Deutsche Philips Industrie GmbH, Aachen Laboratory.

[1]) For details of vacuum-tight seals in *metal* vacuum equipment, see N. Warmoltz and E. Bouwmeester, Philips tech. Rev. **21**, 173, 1959/60 (No. 6).

[2]) See e.g. G. Alma and F. Prakke, Philips tech. Rev. **8**, 289, 1946.

Table I. Data relating to the sealing of glass parts using a glaze (and suitable metals for sealing in these glasses). $\alpha \times 10^7$ denotes the thermal expansion between room temperature and the annealing temperature of the glass; T_w is the highest temperature which the various kinds of glass can withstand without deforming (corresponding to a viscosity of approx. 2.5×10^{12} poise); T_v is the flow point of the glazes (corresponding to approx. 10^4 poise) [3].

	Glasses				Glazes	
	Fused silica and glasses of extremely high SiO_2 (or Al_2O_3) content	Normal hard glass (borosilicate glasses). Suitable metals: tungsten, fernico and molybdenum	Normal soft glass (lime or lead glasses). Suitable metals: platinum, nickel-iron and chrome-iron	Soft glass types with high thermal expansion coefficient (lime and lead glass with high alkali content). Suitable metals: nickel and iron	Zinc-borate glazes	Lead-borate glazes
$\alpha \times 10^7$ (°C^{-1})	6 - 40	35 - 55	85 - 110	115 - 130	36 60	80 - 140
T_w (°C)	1300 - 700	750 - 520	600 - 450	500 - 400		
T_v (°C)					750 - 550	500 - 300

Now it is possible to prepare glazes — on a basis of lead or zinc borate — possessing any desired coefficient of expansion, provided it is not unduly low. In most cases, then, the first condition can be met. A circumstance favourable to the fulfilment of the second condition is that the kinds of silicate glass used for tubes are "long", i.e. they all exhibit a relatively slow drop in viscosity with rising temperature, as required in the usual working procedures, whereas the glazes referred to, not only soften at relatively low temperature, but are "short", i.e. the drop in viscosity with rising temperature is fairly steep.

It becomes difficult to meet both the above conditions for the glaze only when the tube glass combines a low softening point with a low coefficient of expansion ($a = 40$ to 45×10^{-7}), as required for sealing-in tungsten leads, for example. (No suitable glazes are known for types of glass whose thermal expansion coefficient is smaller than this.)

We have tacitly assumed in the foregoing that it is possible, during the sealing or unsealing process, to heat the whole envelope evenly. This is no problem if the tubes are small. The larger the objects, however, and with them the dimensions of the furnace, the more difficult it becomes to apply the process. Moreover, protracted heating at the temperature involved may be harmful to certain components inside the tube.

In such cases the aim will be to confine the heating to the glazed joint and to narrow zones of the glass envelope on either side. The axial temperature gradient of the glass wall in this region must be such that, with the given geometry, the glass will not crack. The temperature distribution in the wall is roughly as shown in *fig. 2*. The arrows indicate

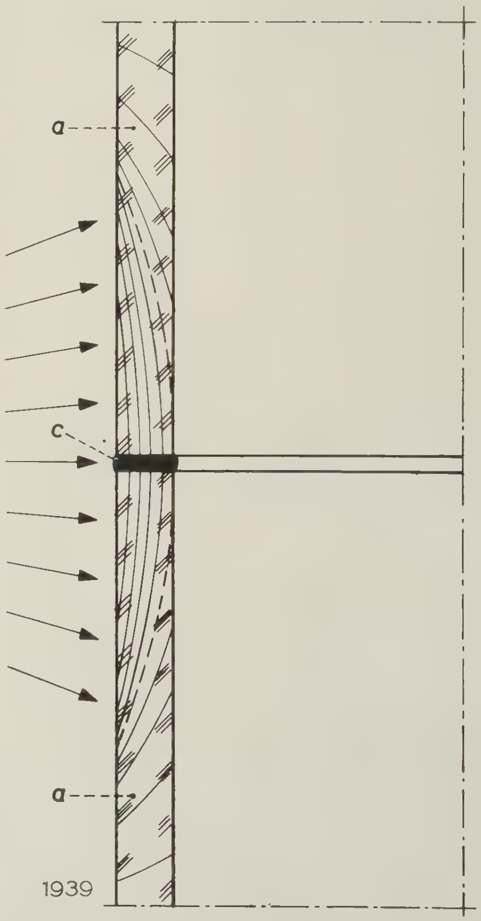

Fig. 2. Longitudinal section of the wall of two cylindrical glass tubes sealed by a glaze. *a* glass wall, *c* glaze. The drawing shows the isotherms inside the glass wall at the moment when the viscosity of the glaze has dropped sufficiently to permit completion of the joint. The isotherm at the flow point is shown by a dashed line. The arrows indicate the applied heat flow.

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[3] See e.g. G. Ch. Mönch, Neues und Bewährtes aus der Hochvakuumtechnik, Knapp, Halle a.d. Saale 1959;
W. H. Kohl, Materials technology for electron tubes, Reinhold, New York 1951;
J. H. Partridge, Glass-to-metal seals, Soc. of Glass Techn., Sheffield 1949.

the direction of the heat flow applied. The kind of heat source is immaterial here, what is important is that when heat is applied externally there must always be a fairly wide belt of high temperature if the flow point is to be reached over the whole width of the joint. Otherwise the joint will not fill out to the edge with molten glaze, resulting in poor bonding. Experiments have shown that the width of the zone subjected to temperature above the flow point should be nearly ten times the wall thickness.

At either side of this hottest zone there are then areas in which the temperature gradually drops to the general temperature of the whole tube, i.e. the temperature to which the tube as a whole may permissibly rise during the sealing or unsealing process. The higher this general temperature may be, the less danger there is of cracking. This is especially important if the tube has an awkward shape, e.g. if it is very compact or has flat surfaces perpendicular to the long axis.

As we have seen, the above principles makes it possible to seal glass parts together and to separate them again without damage and without causing undue softening of the glass. The method is not suitable, however, if there are components near the sealing zone that cannot tolerate either a high temperature gradient or high temperatures.

In this respect the "glazing" technique can be considerably improved if the heat source used for attaining the flow point in the joint is transferred to the joint itself. For this purpose a metal ring, coated on both faces with an appropriate glaze, can be introduced between the parts to be joined and can serve there as a heating element, either by inductive coupling or by the direct passage of current through it. The ring is bonded to the tube walls as the glaze melts, and forms part of the seal when the glass has cooled. The sealing technique evolved from this idea has been investigated on a number of cases in the Aachen laboratory, and will now be discussed.

First, some general observations. The energy supply is stopped as soon as the glaze is judged to be properly fluid, a state which is very soon reached once heating has begun. That is the moment for effecting the necessary displacements, i.e. final alignment when sealing the parts, and pulling apart when unsealing them.

Glazing the ring beforehand, as referred to above, appreciably shortens the period of high temperatures, and thus facilitates the sealing process. In simple cases, however, both operations can be combined.

Here, too, it is obviously an advantage, and indeed often essential, to preheat the complete tubes to a certain general temperature; this accelerates the sealing process and reduces the danger of cracking. Where the tubes are very long it is sufficient, of course, to preheat only the area surrounding the sealing zone. Preheating temperatures should not generally exceed 300 to 350 °C; as mentioned, temperatures higher than this may be damaging to components inside the tube.

The temperature distribution in the tube wall for the new method is roughly as shown in *fig. 3*. The differences compared with fig. 2 are immediately evident: the hottest zone (temperature above the flow point) has shrunk to a narrow strip on either side of the metal ring; the flow temperature can be attained everywhere in the joint at a much lower maximum temperature. An advantage not at once apparent from the figure is that the situation depicted is reached in a matter of a few seconds after switching on the power. This means that the heat supply needed to bring the glaze to the flow point is small, and thus that the dangerous temperature

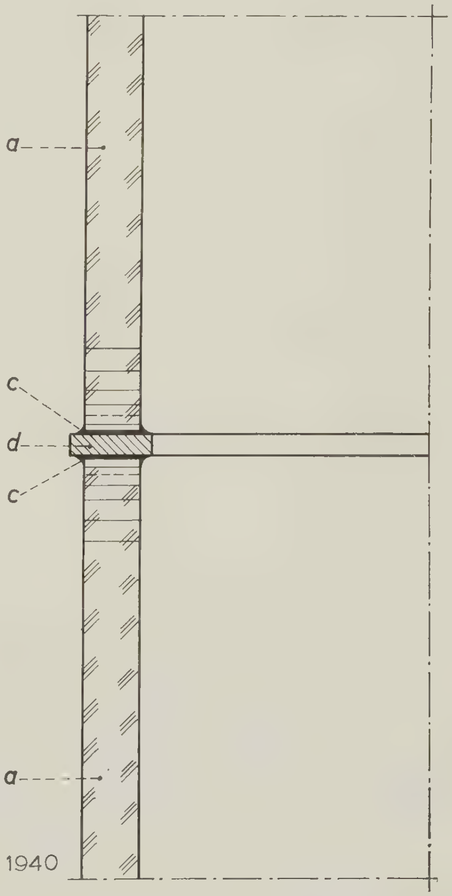

Fig. 3. As fig. 2, but a metal ring is now embodied in the joint. *a* glass wall, *c* glaze, *d* metal ring. The isotherm at the flow point is again a dashed line. The other isotherms are shown in the same steps as in fig. 2.

range is held only for a short time — a particularly important point where a glaze of relatively high flow point has to be used, as for example with types of glass having a low expansion coefficient (e.g. for use with tungsten-wire seals).

Having considered the broad outlines of the method, we can now touch on various particulars, including the choice of materials. As indicated in Table I, for almost every value of expansion coefficient of practical importance in tube manufacture there are several combinations of metal and glass available, each one of a pair matching its partner in expansion. A seal of two such materials, provided it is properly cooled, contains no dangerous stresses and hence has no tendency to break. Most of the sealing metals given in the table are of course also suitable for the heating ring in the method under consideration, the requirements in both cases being the same. Of course, the treatments of the various metals and alloys that may be necessary in practice, to prevent bubble formation, for example, or to improve the adhesion to the glass, are also needed here; they are carried out before the preparatory glazing of the ring.

It has been emphasized that the sealing of glass parts by means of a glaze can succeed only if the materials involved meet fairly stringent requirements as to their behaviour during expansion this being equally valid, as a rule, for glass-to-metal seals. In radio-tube manufacture, however, another technique of glass-to-metal sealing is widely used in which considerable differences in expansion are permissible. In these seals, cracking of the glass is precluded by using a metal part capable of plastic deformation, to compensate for the differences in expansion. This calls for metals of low yield point (see *Table II*) and also for a design that

Table II. Yield points at room temperature of some suitable sealing metals. At higher temperature these values are considerably lower.

Sealing alloys	30 - 50 kg/mm²
Copper (commercial)	approx. 15 kg/mm²
Copper (OFHC)	approx. 6 kg/mm²
Aluminium (99.99%)	approx. 2.7 kg/mm²

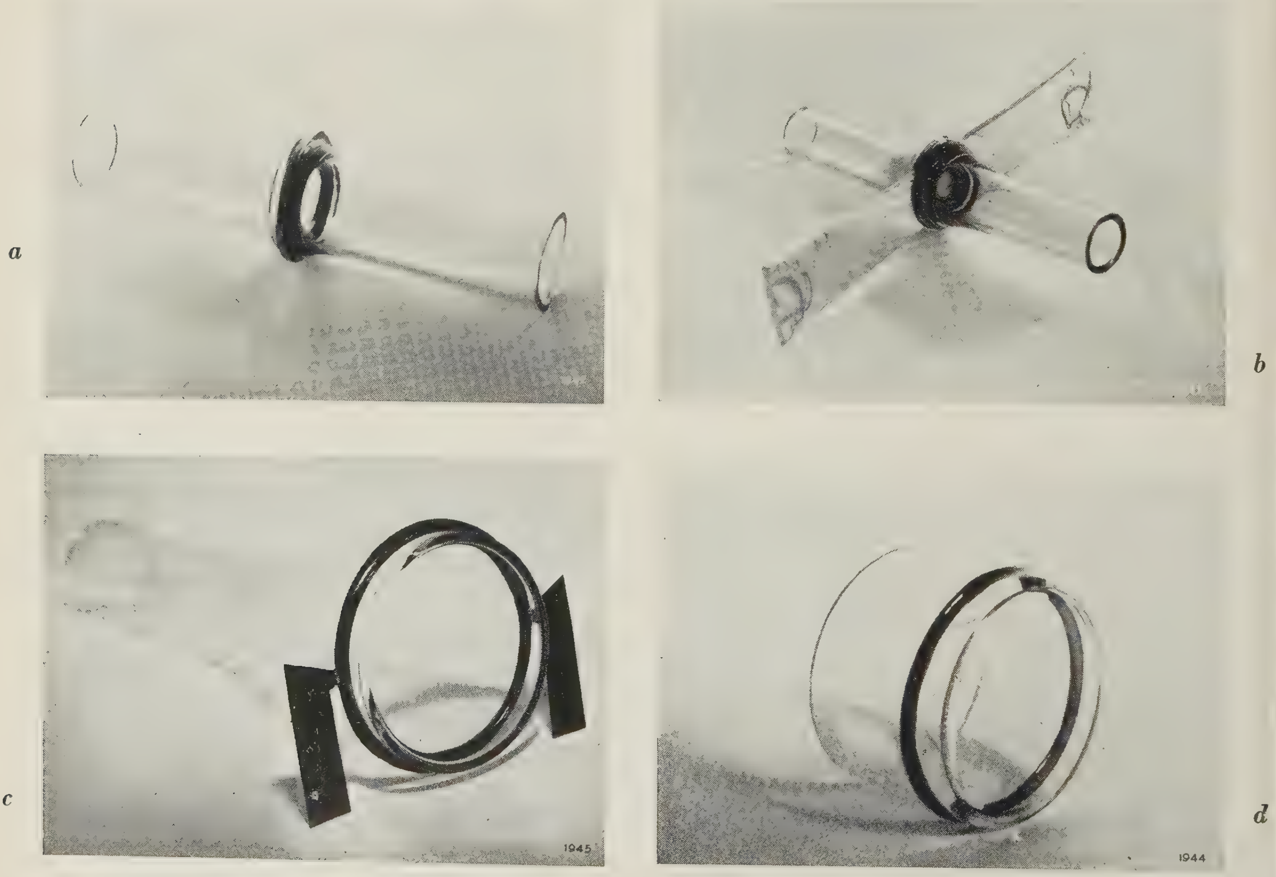

Fig. 4. Some glass high-vacuum seals, made by the method described.
a) Flanged tubes of soft glass, with copper ring.
b) Flanged tubes of soft glass, with aluminium ring. The aluminium "wings" serve as current leads for heating by direct conduction.
c) Seal of glass cap and cone with copper ring and projecting wings for current supply (soft glass).
d) Seal of glass cap and tube with fernico-type alloy (hard glass).

limits the magnitude of the forces produced (small cross-section of metal) and avoids tensile stresses in the glass [4]. Copper, gold and silver can be used with advantage here. If copper is suitably pretreated it bonds excellently with all hard and soft types of glass, provided only that the temperature during the sealing operation remains below the melting point of copper and that strong oxidation can be avoided.

The latter conditions are easily be met when applying the "flowing"-metal principle to the sealing method under consideration. This makes it possible to work at a relatively low temperature and moreover in any desired atmosphere. Indeed, the use of a ring of "flowing" metal in the joints discussed here offers significant practical advantages. To begin with, it adds aluminium to the small number of eligible metals. Aluminum bonds excellently to the type of glaze under consideration, and it also possesses outstanding virtues as a material for use in high-vacuum techniques. Because of its low melting point, however (657 °C), it has not been used hitherto for a direct glass-to-metal seal. This is again the limiting factor: aluminium can be used only for soft types of glass, since the flow points of all known glazes for hard glass are too close to the melting point of aluminium, or may even be higher.

Another advantage is that, if flowing metals are used and the rings are introduced as in fig. 3, the metal insert need not be so very thin as one might expect it to be. If the ring is of pure copper, it can have a thickness from 1 to 1.5 mm; if it is of pure aluminium, a thickness of 2 to 3 mm is permissible. This makes the rings much easier to handle, both mechanically and thermally. The dimensions are even more favourable where cylindrical seals are concerned (according to Housekeeper's method [5]). It will also be plain that the purer, i.e. the softer the metal, the thicker the ring may be. It should be added that rings that are not too thin have the further advantage of making a good vacuum seal possible between glass parts of very different thermal expansion coefficient.

Fig. 4a-d shows a number of glass joints made by the method described.

Soft metals possessing a high coefficient of expansion, differing widely from that of the glass, do have a certain drawback compared with metals and alloys whose expansion matches the glass. The latter allow considerable freedom as regards the shape of the joint, whereas the former demand a construction in which the outside and inside edges are left uncovered, as in fig. 3. Soft metals having a high expansion coefficient are therefore not suitable where — e.g. for reasons of insulation — a complete covering is necessary. Even a local overflow of glaze can easily cause a crack. The reason is evidently that the glass in such a case hinders the plastic flow of the metal. Quite small differences in expansion can then give rise to large forces which, at certain critical places — the glass or the metal itself, or the bond between them — may cause cracks or fracture.

If protruding edges are permitted, a seal as illustrated in *fig. 5* is advantageous, particularly in the case of relatively weak glass walls. The flanges make it possible to use metal rings of comparatively large cross-section (see also fig. 4*a* and *b*).

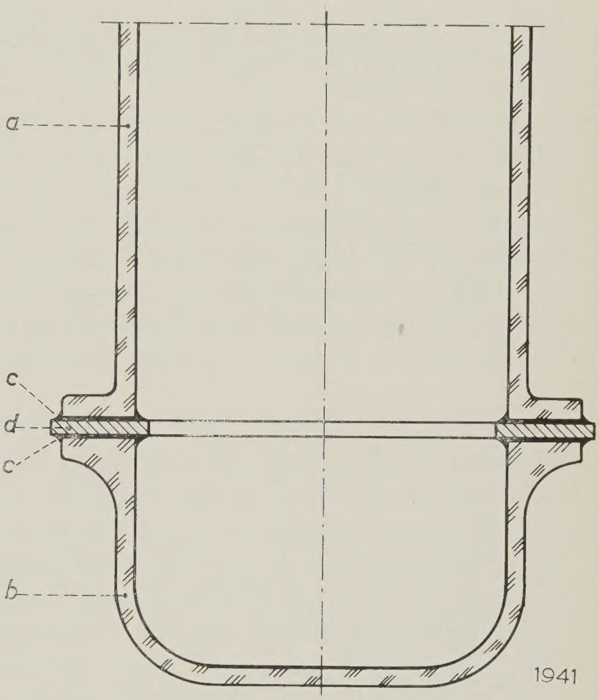

Fig. 5. Tube closed by glass cap. *a* flanged glass tube, *b* flanged glass cap, *c* glaze, *d* metal ring.

The considerations discussed in the foregoing were prompted by the question whether it was possible for the purposes of vacuum technique to make glass joints that would meet various conflicting requirements. They were to be capable of withstanding the necessary pumping temperatures, they should enable the tubes to perform their function after sealing-off, and they should be demountable without seriously damaging them or exposing them to high temperatures. In this brief report of our investigations, it is shown, in general terms, how far these objects can be achieved using glazed joints with the heat source embodied in the joint itself. Details of the method, for example the precise choice of glazes for hard and soft glass, and of alloys for glass-to-metal seals for the various types of hard glass, are, however, outside the scope of this article.

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[4] See J. L. Ouweltjes, W. Elenbaas and K. R. Labberté, Philips tech. Rev. **13**, 109, 1951/52, in particular fig. 7, and B. Jonas, Philips tech. Rev. **3**, 119, 1938.

[5] See the book by Partridge referred to under [3], p. 160 *et seq.*, and that by Kohl, p. 60 *et seq.*

Summary. The method described here of making vacuum-tight demountable glass seals uses a metal ring coated on both sides with a suitable glaze. When the ring is heated, e.g. inductively, the glaze melts and the preheated glass parts are then pressed together. Embodying the heat source in the joint results in a favourable temperature distribution, the zone of highest temperature (and steep temperature gradient) being restricted to the immediate vicinity of the joint. Moreover the amount of heat supplied and the heating time are reduced to a minimum. As the metal ring remains in the joint, it can later serve again as local heat source if the joint is to be unsealed, without damaging the parts. A further advantage when the ring is of a soft metal is that, being capable of plastic deformation, it allows fairly wide disparities between the thermal expansion coefficients of the glass parts joined.

ABSTRACTS OF RECENT SCIENTIFIC PUBLICATIONS BY THE STAFF OF N.V. PHILIPS' GLOEILAMPENFABRIEKEN

Reprints of these papers not marked with an asterisk * can be obtained free of charge upon application to Philips' Electrical Ltd., Century House, Shaftesbury Avenue, Londen W.C. 2, where a limited number of reprints are available for distribution.

2738: H. B. G. Casimir, J. Smit, U. Enz, J. F. Fast, H. P. J. Wijn, E. W. Gorter, A. J. W. Duyvesteyn, J. D. Fast and J. J. de Jong: Rapport sur quelques recherches dans le domaine du magnétisme aux Laboratoires Philips (J. Phys. Radium **20**, 360-373, 1959, No. 2/3). (Report on various researches at Philips in the field of magnetism; in French.)

In the first part of the paper the crystalline anisotropy of a number of hexagonal oxidic compounds containing barium is discussed. In the absence of an external magnetic field the magnetization vector can point in an arbitrary direction with respect to the c-axis. This behaviour can be described with two anisotropy constants. Examples are given of materials with a preferential direction (c-axis), with a preferential plane (basal plane) as well as with a preferential cone for the magnetization vector. The latter case occurs at relatively low temperatures in crystals containing cobalt. There are also materials in which, at different temperatures, all three types of anisotropy occur. The relatively weak anisotropy in the basal plane, which has six-fold symmetry, has been measured. In crystals having only trivalent metal ions, two such ions can be replaced by one divalent and one quadrivalent ion. It appears that substitution of cobalt again promotes the occurrence of a preferential plane of the magnetization, as in the oxides which contain divalent metal ions. The classical dipole-dipole energy has been computed and it is shown that it can account for the observed anisotropy in the structure containing two successive barium layers, which, although not containing cobalt, shows a preferred plane for the magnetization vector. The anisotropy in the structure containing single barium layers, which has a preferred direction of the magnetization vector, is not explained by this mechanism, and presumably originates from spin-orbit interaction. The influence of controlled precipitation on the magnetic properties of alloys is discussed in the last section. With the aid of an electron microscope it is shown that a precipitate, consisting of long parallel needles in the optimal case, causes the high $(BH)_{\max}$ value (up to 12×10^6 gauss-oersteds) of single crystal "Ticonal" ("Alnico") containing 34% cobalt, that has undergone a special heat treatment in a magnetic field. It is further shown that a (110) [001] texture can be obtained in 3%-silicon iron only if the metal contains a precipitate of favourable composition (e.g. Si_3N_4 or MnS) and division.

2739: F. L. H. M. Stumpers: Interpretation and communication theory (Synthese **11**, 119-126, 1959, No. 2).

This article (contribution to a symposium held in 1954) describes in brief the analogy between the interpretation of translated texts and the interpretation of messages transmitted over noisy transmission channels. Also discussed are some of the statistical methods from communication theory that have been applied for the quantitative study of literature and language.

2740: P. M. Cupido: Some views on automatic control in glass factories (Glastech. Ber. **32 K**, I/1-I/5, 1959, No. 1).

A brief outline is given of some different modes of automatic control and the stability of the controlled system. The application of automatic control in glass processes is discussed. It is no remedy for bad furnace design or bad process conditions, but it will give a sound background for studying the process and will make it possible to evaluate improvements. If the optimum settings for the improved process have been found, automatic control will be the most reliable means of keeping the process conditions within the necessary limits.

2741: A. M. Kruithof and A. L. Zijlstra: Different breaking-strength phenomena of glass objects (Glastech. Ber. **32 K**, III/1-III/6, 1959, No. 3).

Survey of the mechanical strength of various glass objects. Not only thin glass fibres but also massive glass objects can have very high strengths (200 kg/mm^2) if care is taken that the surface is not damaged. From this it is concluded that the structure has only a secondary effect on the strength. A kind of "strength scale" is constructed showing how various combinations of structure and surface effects give rise to various levels of strength.

2742: J. Goorissen, F. Karstensen and B. Okkerse: Growing single crystals with constant resistivity by floating-crucible technique (Solid state physics in electronics and telecommunications, Proc. int. Conf., Brussels, June 2-7, 1958, edited by M. Désirant and J. L. Michiels, Vol. 1, pp. 23-27, Academic Press, London 1960).

See Philips tech. Rev. **21**, 185-195, 1959/60 (No. 7).

2743: J. H. Uhlenbroek and J. D. Bijloo: Investigations on nematicides, II. Structure of a second nematicidal principle isolated from Tagetes roots (Rec. Trav. chim. Pays-Bas **78**, 382-390, 1959, No. 5).

A second nematicidal principle isolated from *Tagetes* roots has been identified as 5-(3-buten-1-ynyl)-2,2′-bithienyl. Upon catalytic hydrogenation 5-butyl-2,2′-bithienyl was obtained which by comparison of infrared spectra proved to be identical with a synthetically prepared sample. The 5-(3-buten-1-ynyl)-2,2′-bithienyl was further characterized by its infrared absorption.

2744: H. F. Hameka: Berechnung der magnetischen Eigenschaften des Wasserstoffmoleküls (Z. Naturf. **14a**, 599-602, 1959, No. 7). (Calculation of the magnetic properties of the hydrogen molecule; in German.)

To calculate the proton screening and the magnetic susceptibility of the hydrogen molecule, wave functions are introduced built up from calibration-invariant atomic wave functions. The calculation is done for two cases, the ground-state wave function being described either by the Wang or the Rosen approximation. In the first case the nucleus screening constant turns out to be 2.631×10^{-5} and the susceptibility -3.920×10^{-6}. In the second case the values are 2.732×10^{-5} and -4.045×10^{-6}, respectively.

2745: N. W. H. Addink: Note on the analysis of small quantities of material by X-ray fluorescence (Rev. univ. Mines **102**, 530-532, 1959, No. 5).

One point of particular importance in X-ray fluorescence analysis is the so-called inter-element effect: fluorescent radiation originating in one element A in a sample is partially absorbed by an element B, to an extent depending on the concentration of B. In the present investigation it is shown that this spurious effect can be eliminated by a) the use of only dilute solutions, b) the use of thin layers of powdered materials (sample required less than 1 mg). With method (b) systematic errors are held to within 4%.

2746: G. Klein and J. M. den Hertog: A sine-wave generator with periods of hours (Electronic Engng. **31**, 320-325, 1959, No. 376).

By means of an inverse-function generator it is possible to derive a triangular voltage accurately from a sinusoidal one. By applying negative feedback the reverse can also be achieved. Making use of this possibility an ultra-low frequency sine-wave generator was designed for maximal periods of 3½ hours. The distortion is then negligibly small. If a slight distortion is permissible, this period can be increased considerably. An important feature of this generator is the fact that no transient phenomena occur. The inverse-function generator can also be used for various other purposes, one of them being a logarithmic voltmeter covering the range from approximately 10 mV to some tens of volts.

2747: C. J. M. Rooymans: A new type of cation-vacancy ordering in the spinel lattice of In_2S_3 (J. inorg. nucl. Chem. **11**, 78-79, 1959, No. 1).

In_2S_3 has a crystal structure closely similar to the spinel structure: the formula can be written $In_{8/3}\square_{1/3}S_4$, analogous to spinel AB_2O_4 or AB_2S_4. In contrast to γ-Fe_2O_3, In_2S_3 has cation vacancies in certain tetrahedral positions; these positions are ordered, giving rise to a superlattice.

2748: D. J. Kroon, C. van de Stolpe and J. H. N. van Vucht: Etude de la résonance nucléaire magnétique de l'hydrogène inclus dans l'alliage Th_2Al (Archives des Sciences **12**, fasc. spéc., 156-160, 1959). (Nuclear magnetic resonance study of hydrogen in Th_2Al; in French.)

Per molecule in the crystal of Th_2Al there are four sites capable of accommodating hydrogen atoms. If only two of these sites are occupied by protons (Th_2AlH_2), the diffusion rate of hydrogen in the crystal is high at room temperature, resulting in a narrow resonance line. Below 100 °K this motion ceases and a broad resonance line results. From the temperature dependence of the line width it is found that the diffusion activation energy is 0.22 eV. Similar measurements have been made for Th_2AlH_3. From the shape of the resonance line in this case it is concluded that there is a certain equilibrium between "free" and "bound" hydrogen. If proton motion is hindered because all interstitial sites are filled (Th_2AlH_4 and $Th_2AlH_2D_2$), the line is broad even at room temperature. (See also Philips tech. Rev. **21**, 297-298, 1959/60, No. 10.)

2749: J. S. van Wieringen and A. Kats: Paramagnetic resonance of hydrogen in fused silica (Archives des Sciences **12**, fasc. spéc., 203-204, 1959).

Pure fused silica shows neither optical absorption nor paramagnetic resonance after irradiation with X-rays at room temperature. On the other hand, irradiation at the temperature of liquid nitrogen produces two absorption bands in the ultraviolet and a paramagnetic resonance spectrum whose intensity grows with the percentage water present. Paramagnetic resonance measurements suggest that the colour centres responsible for these effects are hydrogen atoms. They disappear after a few minutes at a temperature of 10-20 °C above that of liquid nitrogen.

2750: J. Davidse and B. T. J. Holman: A suppression filter with variable bandwidth (T. Ned. Radiogenootschap **24**, 199-209, 1959, No. 4).

This paper deals with the design of a notch filter with variable bandwidth. The loading capacitance is neutralized by means of a feedback circuit; in addition with this circuit negative load resistances can be realized. In this way very small bandwidths can be obtained. It is shown that bandwidth variation can be obtained by variation of the loading resistance. The transient response and the overshoot of the filter are calculated. Finally the practical circuit is given and discussed briefly.

2751: G. D. Rieck: Rekristallisation von Wolframdrähten (Hochschmelzende Metalle, 3rd Plansee-Seminar, Reutte/Tirol, June 22-26, 1958, edited by F. Benesovsky, pp. 108-119; published 1959). (Recrystallization of tungsten wires; in German.)

Tungsten for use in incandescent filaments is provided with a "dope" which promotes the growth of large crystals during recrystallization. These crystals show a fragmentation structure, in particular after bending, from which it can be concluded that the residues of the dope are present as filaments parallel to the wire axis. These large crystals have an orientation — the [531] direction lies in the wire axis — that differs from the texture of the small crystals of pure tungsten and which appears to depend on the action of the dope. The occurrence of this particular orientation should not be attributed to a deviation from the drawing texture, but can be explained by two facts. Firstly this orientation is able to survive the glide process occurring during deformation of the crystallites, whereas elsewhere a [110] texture arises. Secondly the damaged walls of impurities inhibit the growth of these grains less than that of others.

This interpretation of crystal growth also provides an explanation of the observed fragmentation phenomena.

2752: J. M. Stevels: L'évolution de la technologie et de la recherche verrière depuis la guerre (Vetro e Silicati **3**, No. 14, 23-30, 1959). (Glass technology and research since the war; in French.)

The author demonstrates that the development of glass technology and research since the war has been extraordinary. The paper includes the following three sections: 1) basic research on glass, 2) manufacture of glass objects, 3) improvements in glass. It is interesting to note that there have been two different trends recently in the technology of glass: 1) realization that partially crystallized glass has particularly attractive properties, 2) realization that very often it is the finishing of the moulded glass that makes its excellent properties evident.